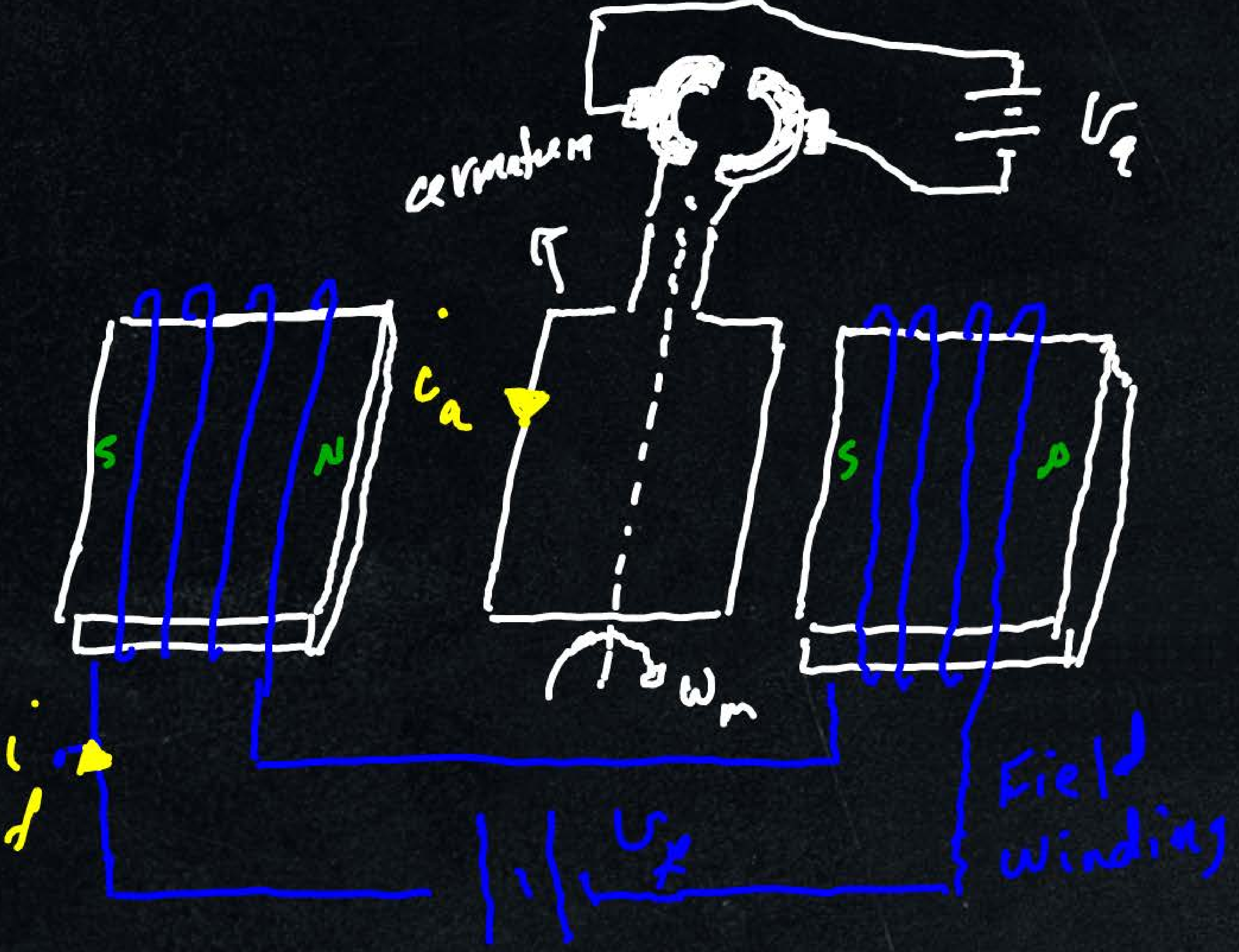
DC Machine Drive

[Theory of operation of DC Motor]



Electro-magnet force:

$$F_e = i_a \vec{l} \times \vec{\underline{B}}$$

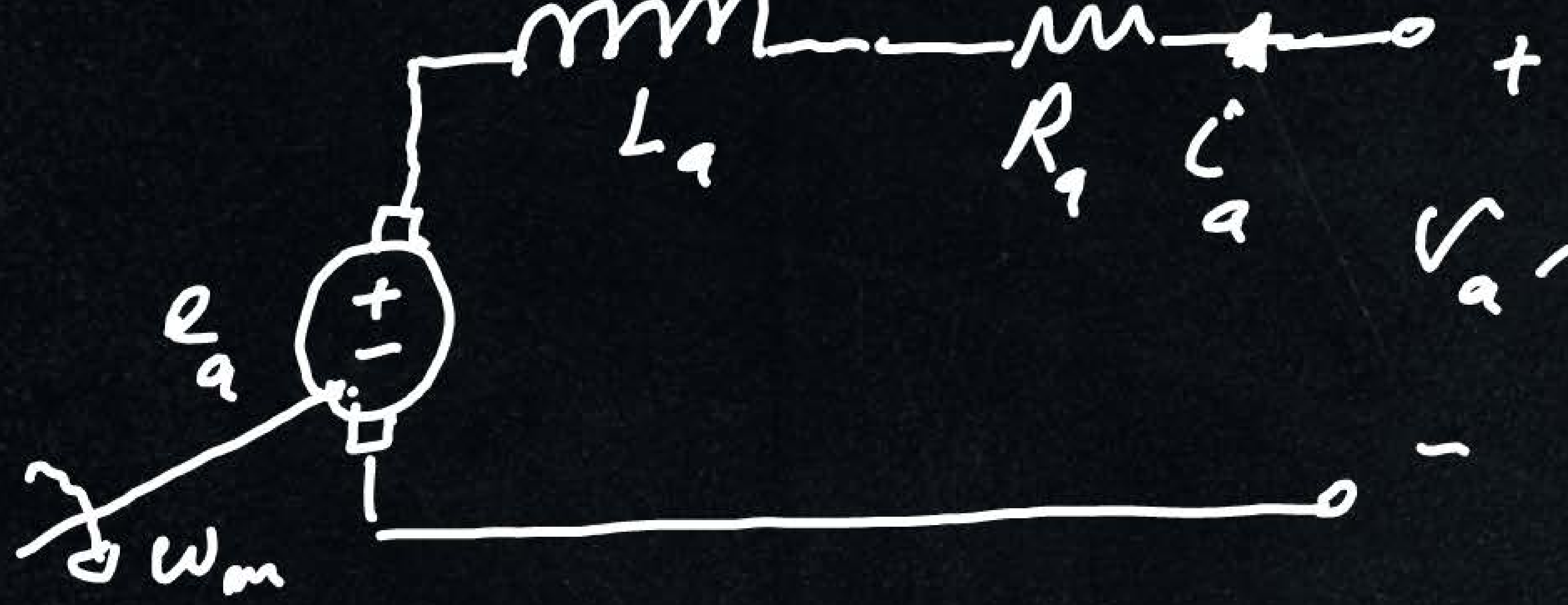
Electro-magnet torque:

$$T_{e} = \vec{r} \times \vec{F_e}$$

Operation of DC Motor

- The electric current, i_a, passes through the armature winding via a commutator.
- When i_a passes through the armature winding in a magnetic field, a magnetic force, F_e, is induced ($\vec{F}_e = i_a \vec{l} \times \vec{B}$).
- The magnetic force produces a torque, which turns the DC motor.

Equivalent circuit of DC motor armature :-



$V_a \rightsquigarrow$ terminal DC voltage

R_a & L_a are the resistance & self-inductance of armature winding

e_a: Back emf voltage or induced voltage

$$e_a = K\phi\,\omega_m$$

$$e_a = K \Phi_f \omega_m$$

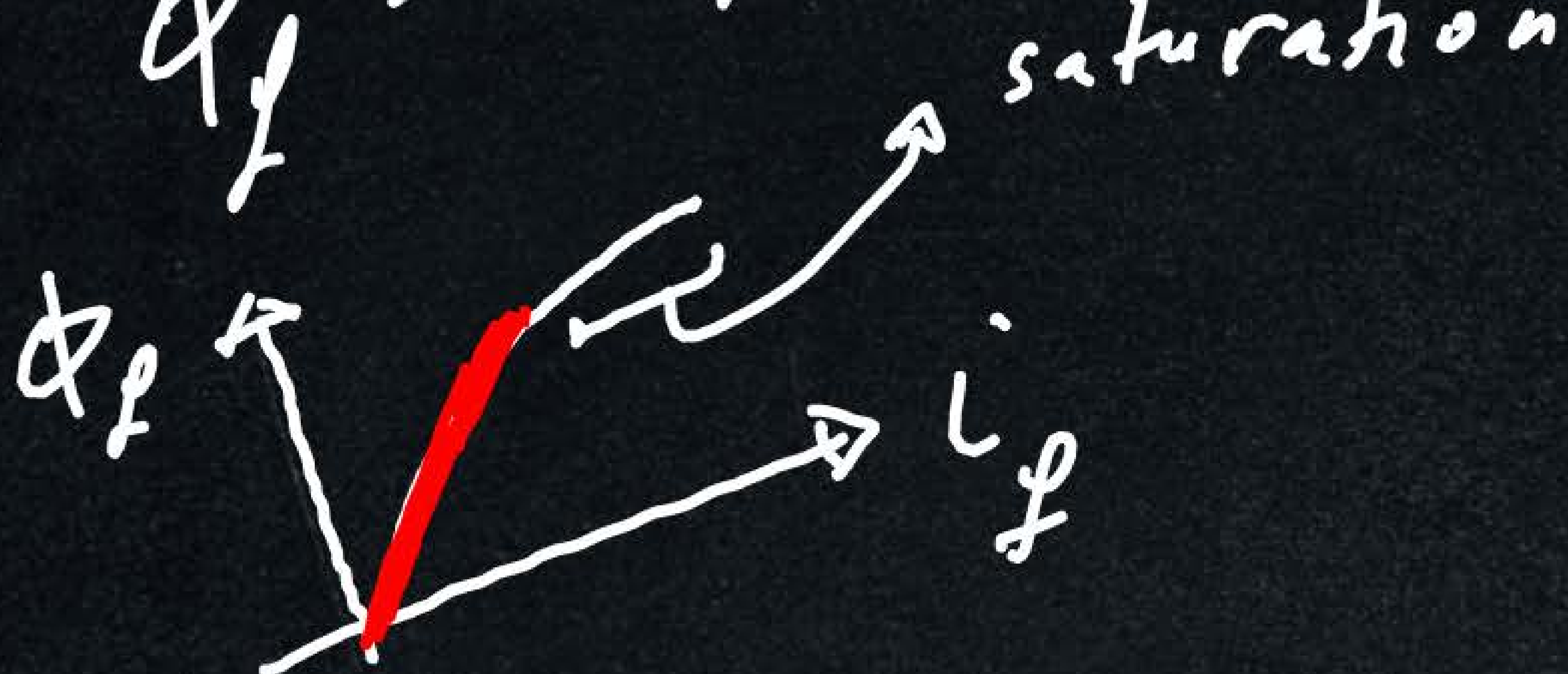
induced voltage

rotor's speed

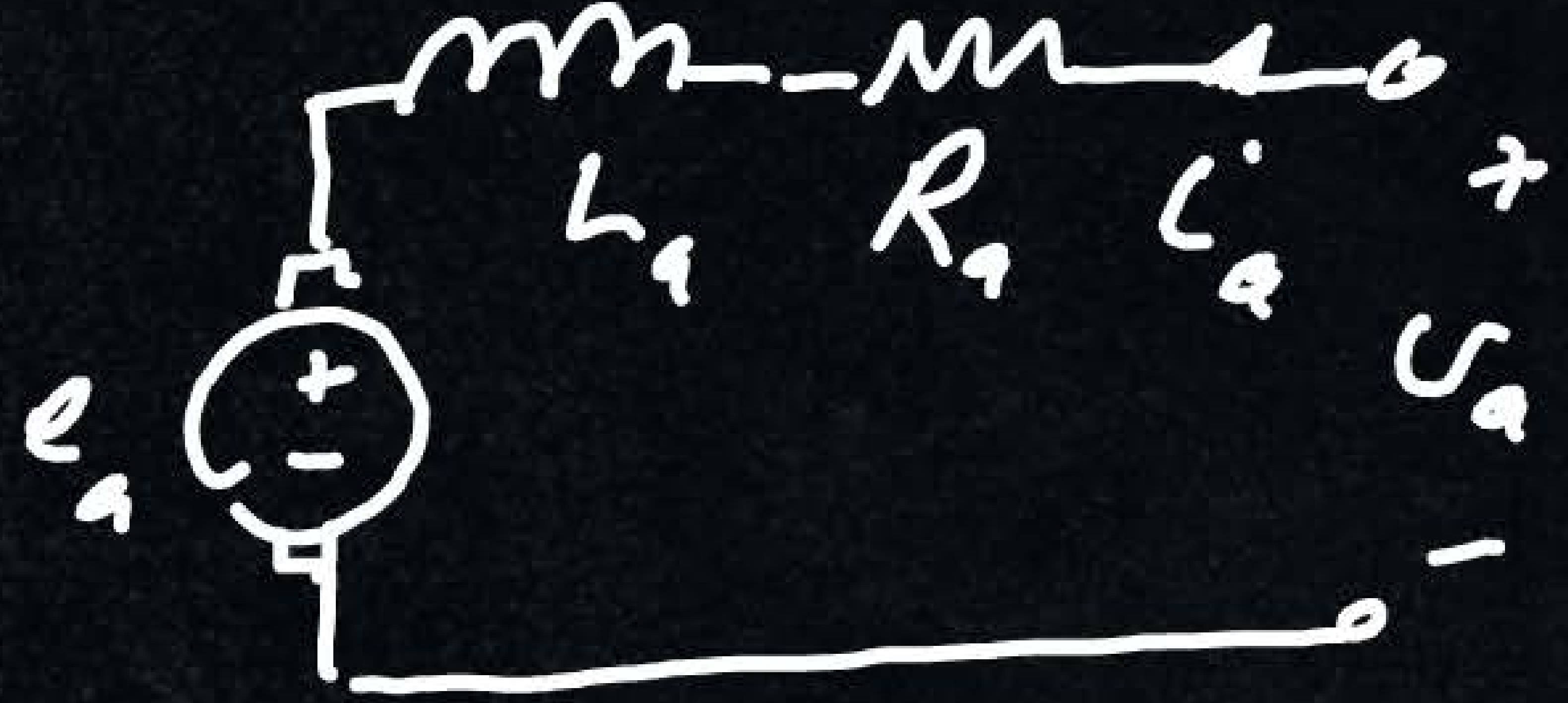
constant depends on the machine structure

machine's flux

$$\Phi_f = c\, i_f$$



Torque Equation



KVL in the armature circuit :-

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a$$

under ss condition

$$\rightarrow \frac{di_a}{dt} = 0$$

$$\Rightarrow v_a = R_a i_a + e_a$$

$$V_a = R_a i_a + e_a$$

$$\underbrace{V_a i_a}_{\text{Total input power}} = \underbrace{R_a i_a^2}_{\text{Armature copper losses}} + \underbrace{e_a i_a}_{\text{Air-gap power}}$$

Air-gap power

"Effective power that has been transformed to mechanical power"

$$P_a = e_a i_a = (K\phi_f \omega_m)\, i_a$$

$$P_a = e_a i_a = (K\Phi_f \omega_m)\, i_a$$

$$P_a = T_{el}\, \omega_m$$

$$T_{el} \cancel{\omega_m} = (K\Phi_f \cancel{\omega_m})\, i_a$$

$$T_{el} = K\Phi_f\, i_a$$

Machine Equations (time-domain)

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a$$

$$e_a = K\phi_f \omega_m = K' \omega_m \quad \text{"constant flux"}$$

$$T_{el} = K\phi_f i_a = K' i_a \quad \text{"constant flux"}$$

$$T_{el} = T_L + J\frac{d\omega_m}{dt} \quad \text{"Newton's } 2^{nd} \text{ Law"}$$

$$v_a \xrightarrow{\mathcal{L}} V_a$$

Machine Equations (s-domain)

$$V_a = (R_a + sL_a)\, I_a + E_a$$

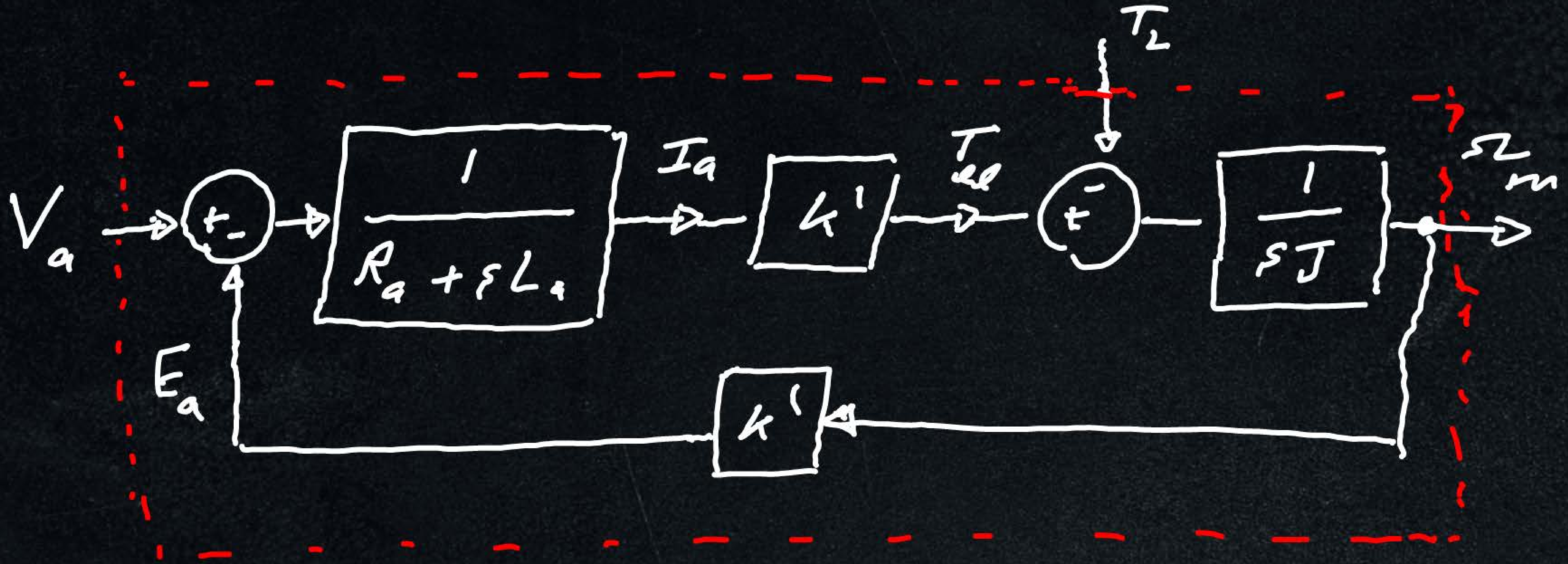
$$E_a = k' \Omega_m$$

$$T_{el} = k' I_a$$

$$T_{el} = T_L + (sJ)\, \Omega_m .$$

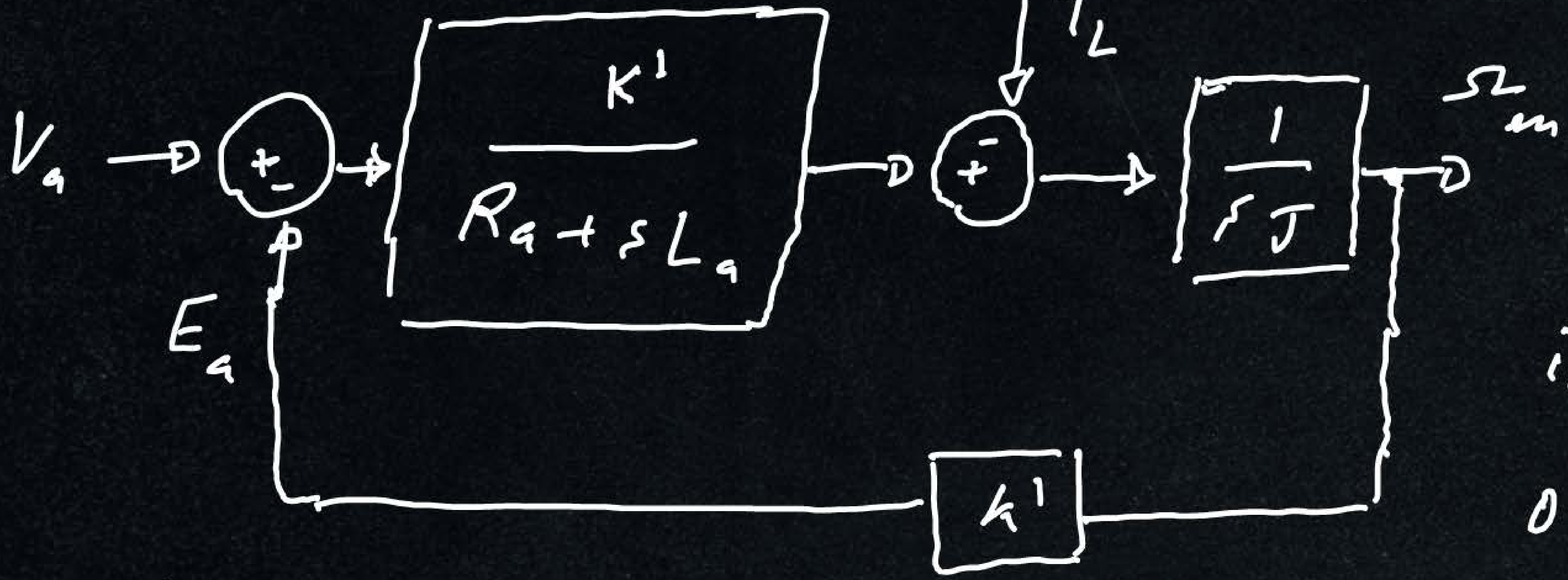
$$V_a = (R_a + sL_a) I_a + E_a, \quad E_a = k' \Omega_m,$$

$$T_{el} = k' I_a, \quad T_{el} = T_L + sJ\Omega_m$$



Block Diagram of DC Motor

Block Diagram and Transfer Function



Linear system

inputs: V_a, T_L

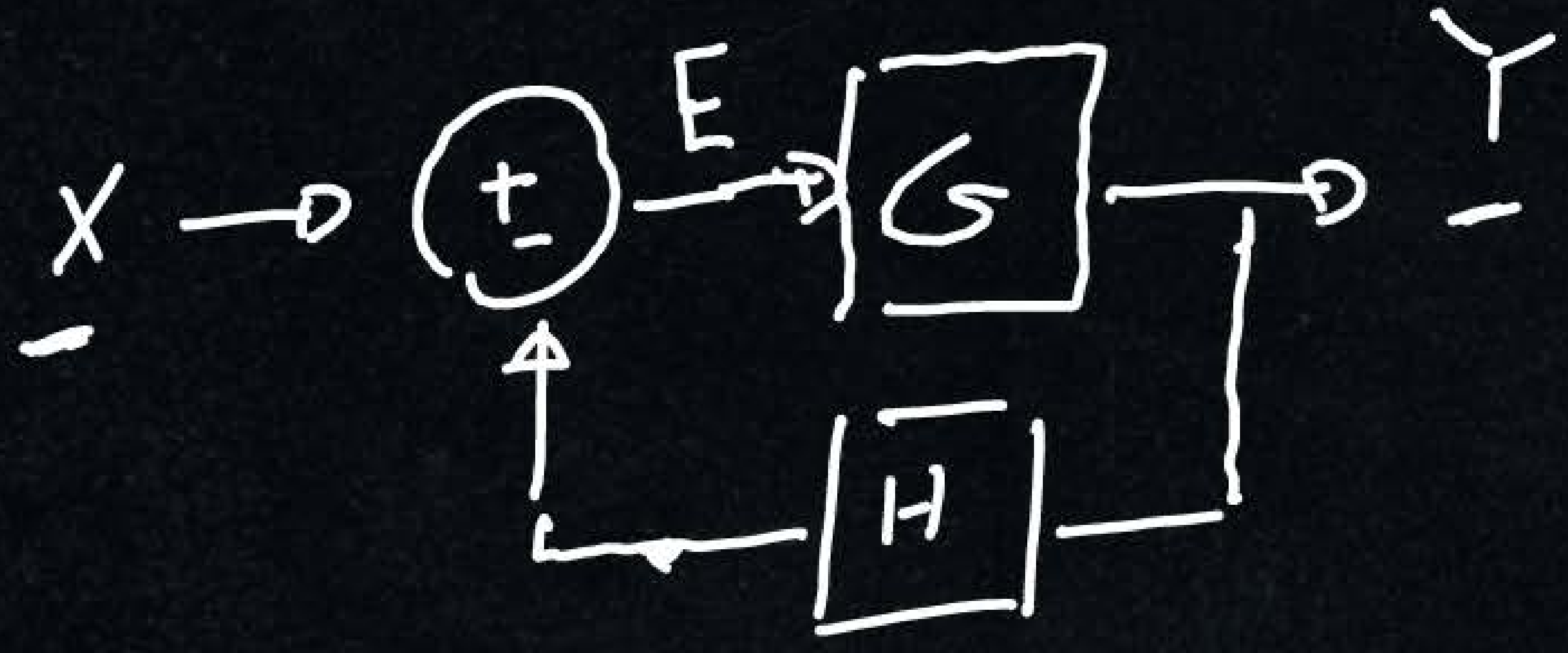
output $= \Omega_m$

$$\Omega_m = T_1 V_a + T_2 T_L$$

where $T_1 = \Omega_m / V_a$ when $T_L = 0$

$T_2 = \Omega_m / T_2$ when $V_a = 0$

Closed loop system



G : Direct transfer function

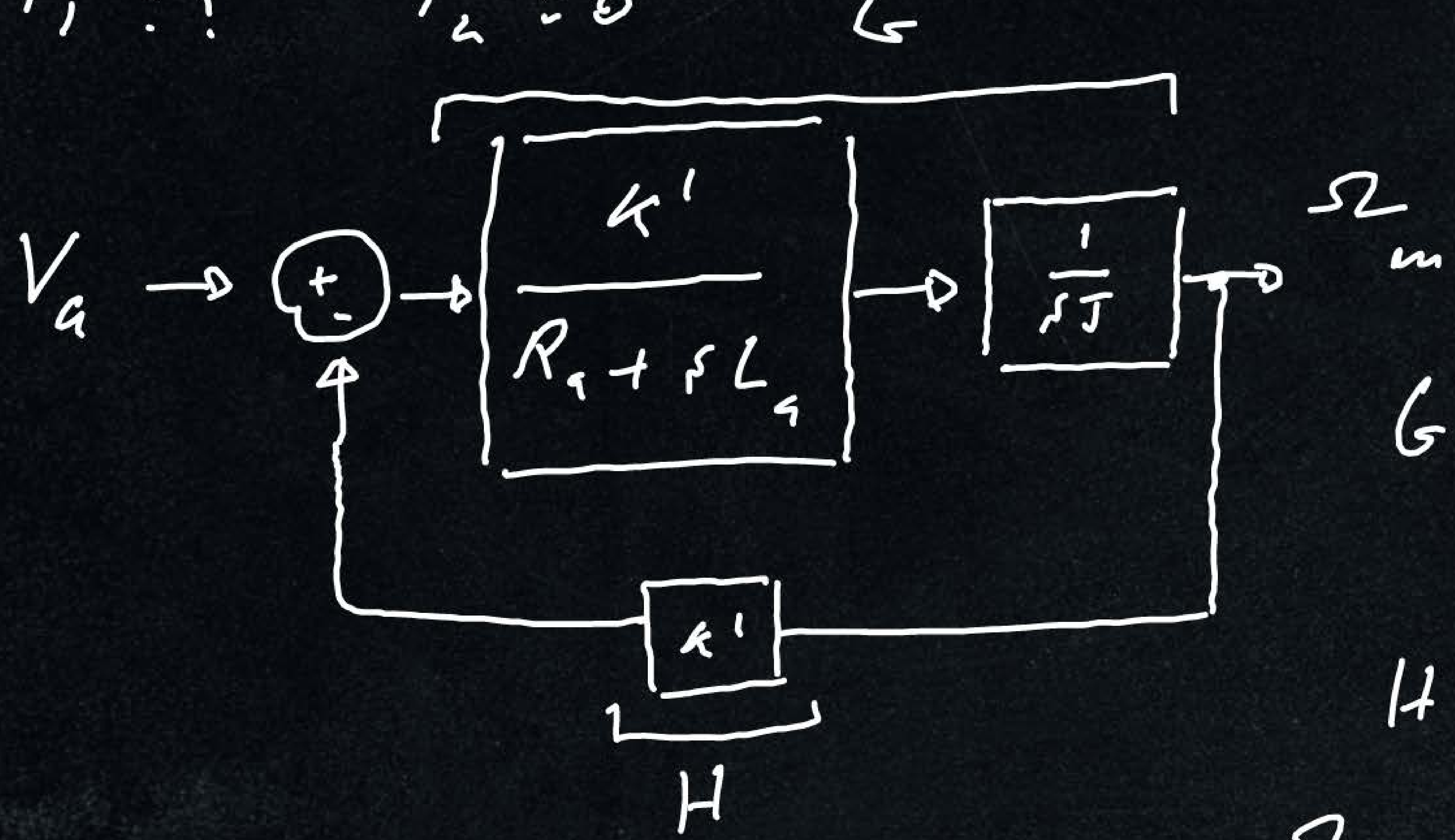
H : Feedback transfer function

$$Y = GE = G(X - HY)$$

$$Y = GX - GHY$$

$$\Rightarrow \frac{Y}{X} = \frac{G}{1 + GH}$$

T_1 ?? $T_2 = 0$

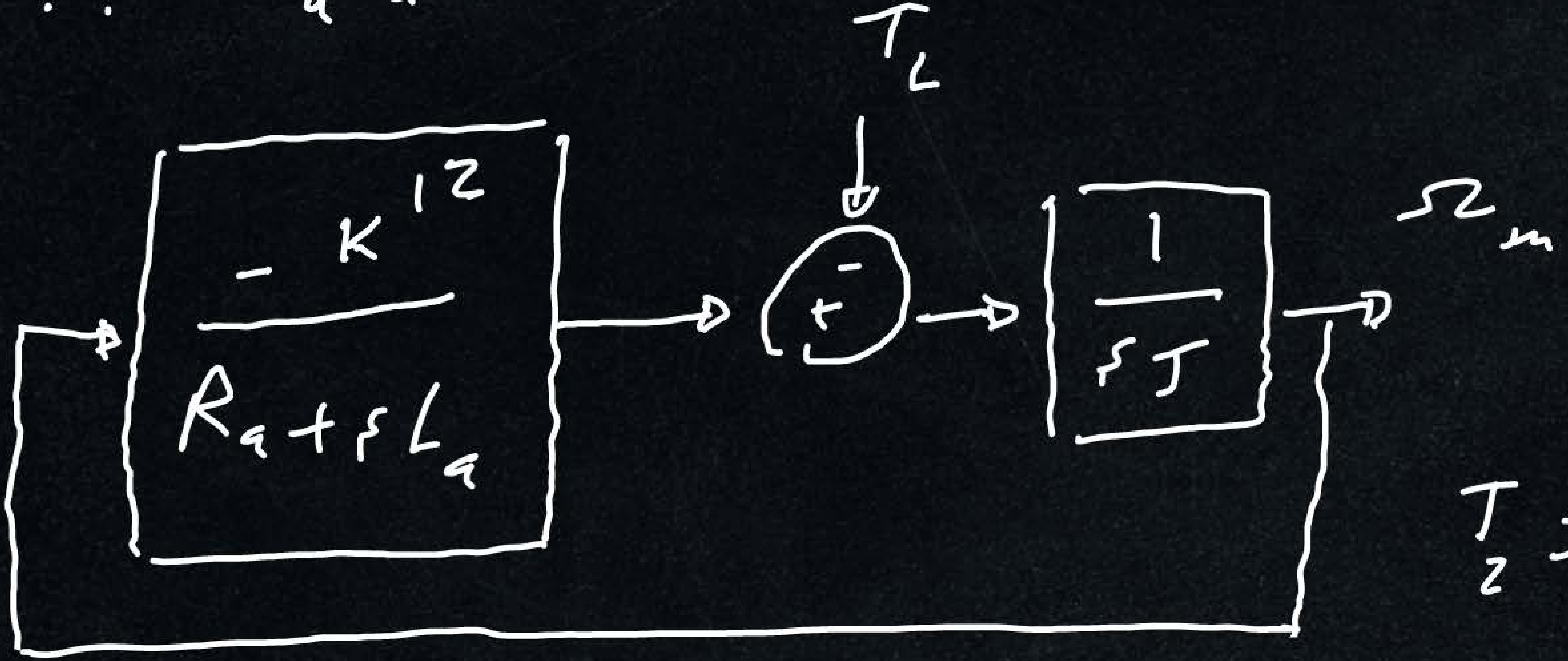


$$G = \frac{k'}{sJ\left[R_a + sL_a\right]}$$

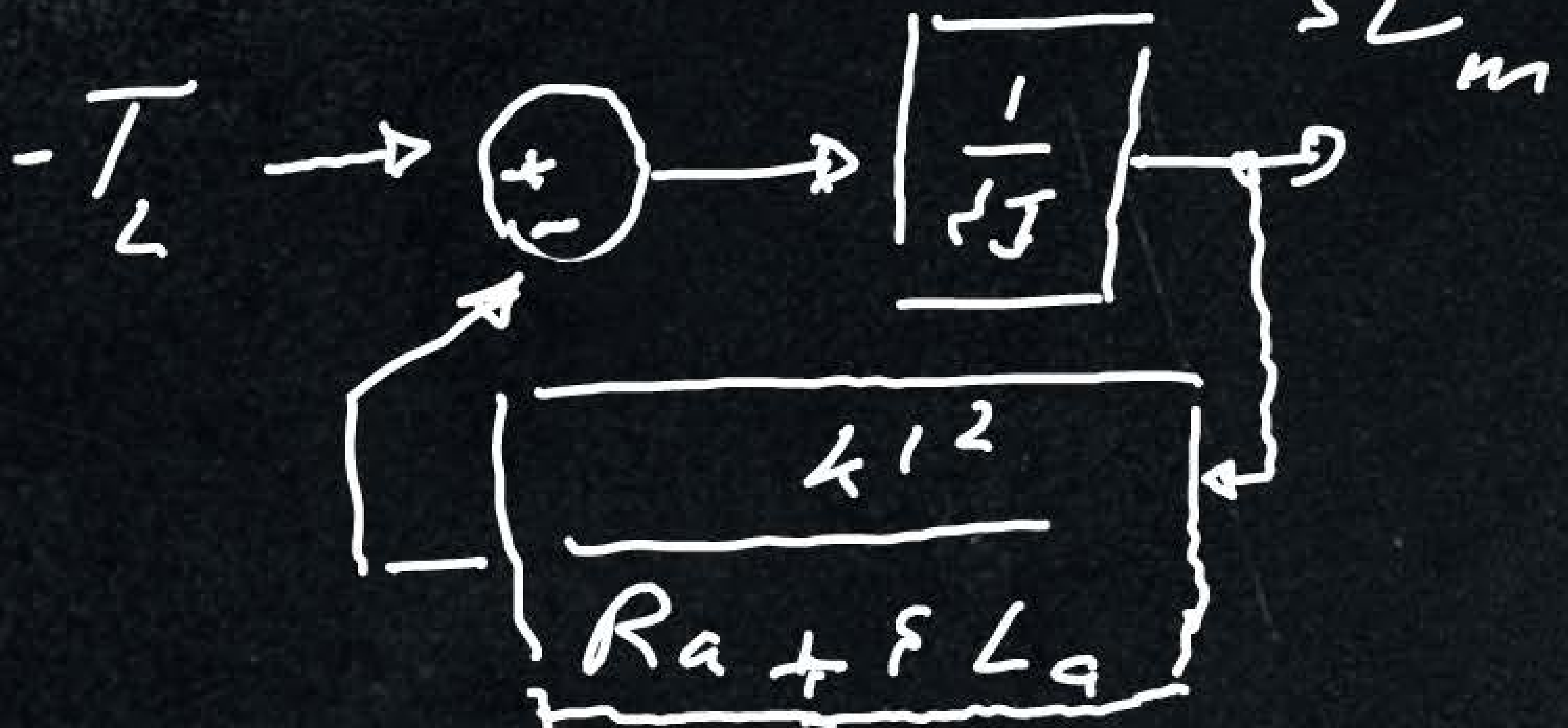
$$H = k'$$

$$T_1 = \frac{\Omega_m}{V_a} = \frac{G}{1 + GH}$$

T_2 ?? $V_a = 0$



$$T_2 = \frac{-G}{1 + GH} = \frac{\Omega_m}{T_L}$$



$$G = \frac{1}{sJ}$$

$$H = \frac{k'^2}{R_a + sL_a}$$

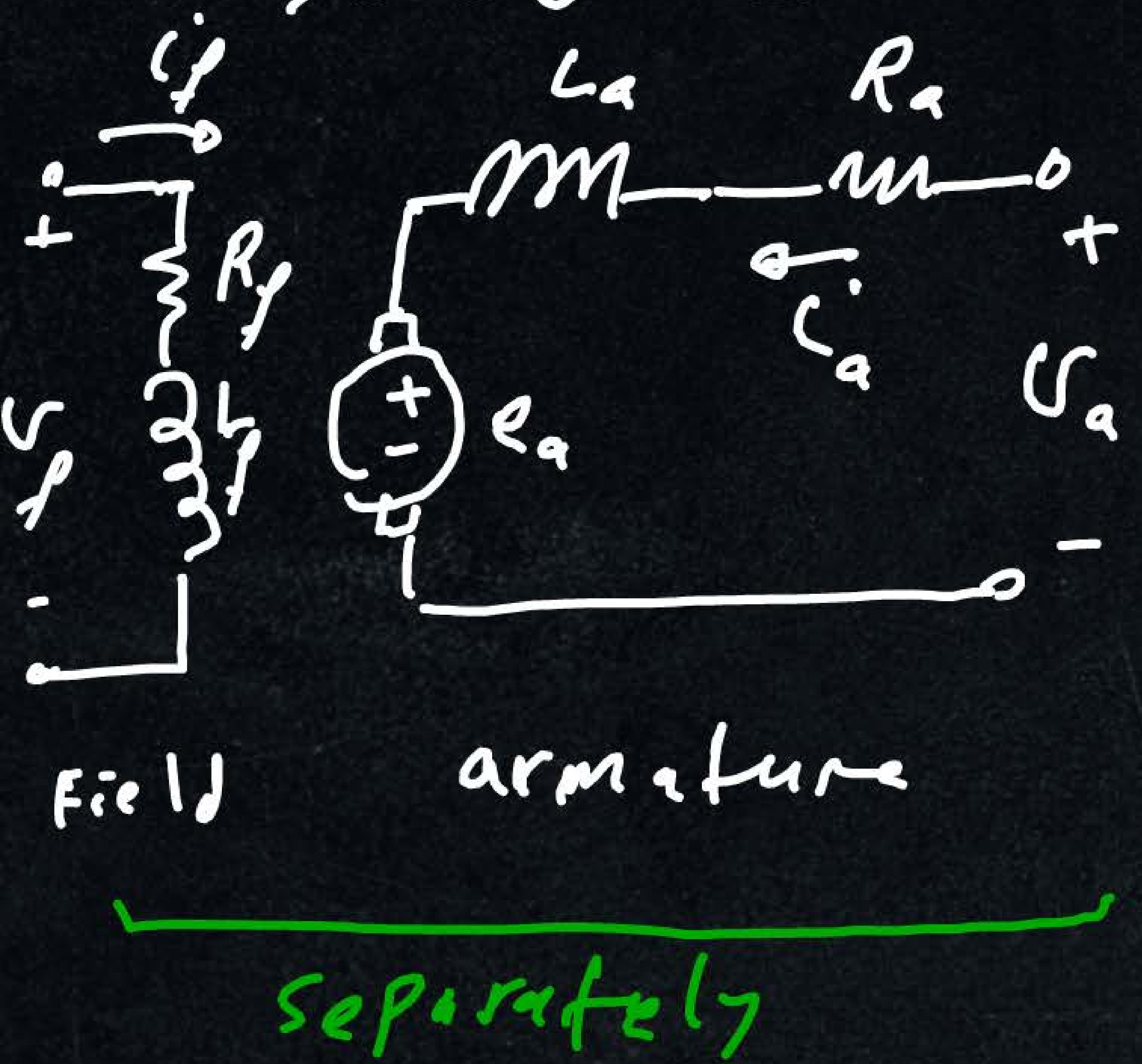
$$\Omega_m = T_1 V_a + T_2 T_L$$

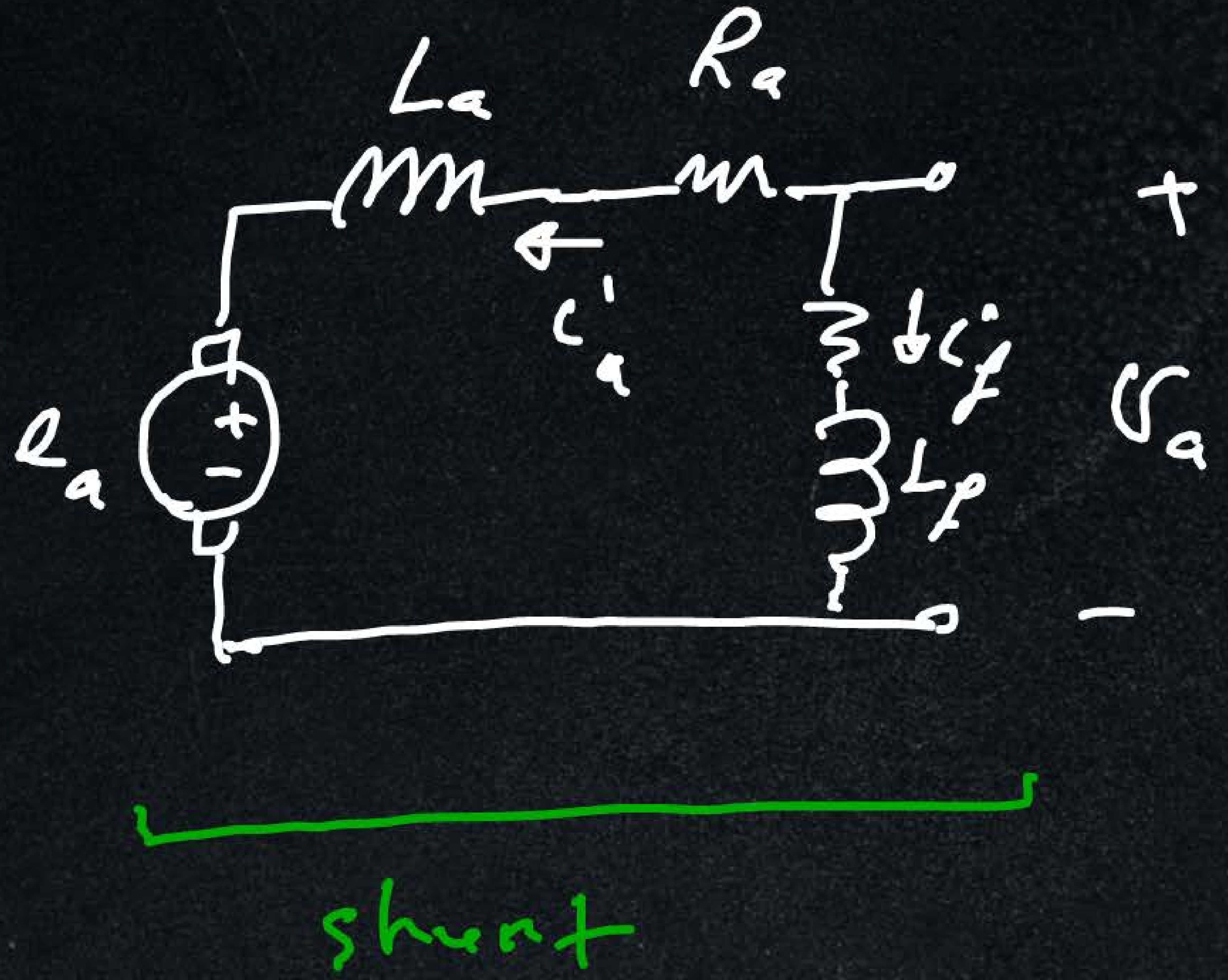
Field Excitation

- separately excited DC machine
- shunt excited DC machine
- series excited DC machine
- compound DC machine
- permanent magnet DC machine (PMDC)

Steady state speed - torque relations

① separately & shunt





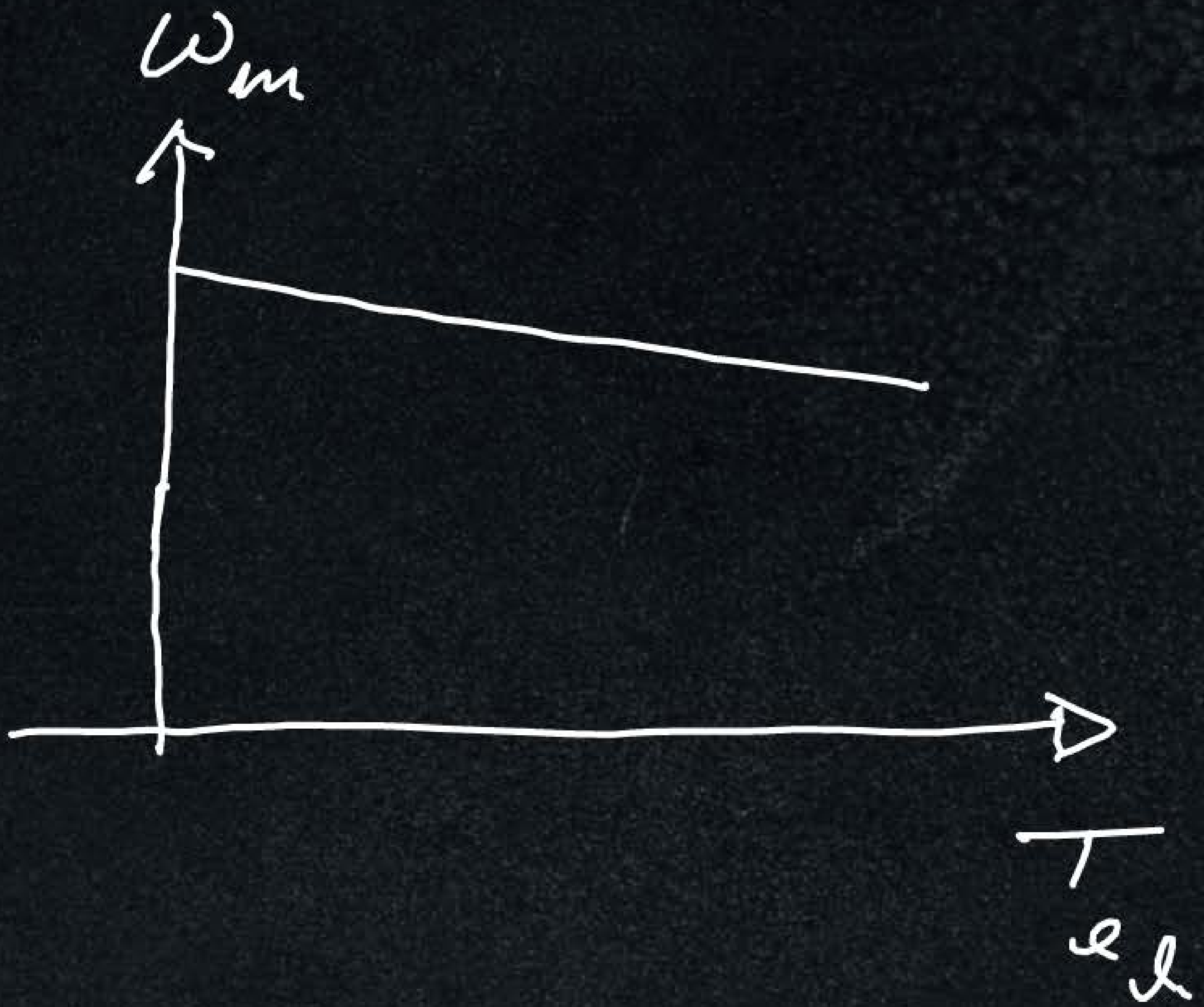
$$V_a = R_a i_a + k\phi_f \omega_m \text{ ---- } (1)$$

$$T_{el} = k\phi_f i_a \Rightarrow i_a = \frac{T_{el}}{k\phi_f} \text{ --- } (2)$$

$$(2) \rightarrow (1)$$

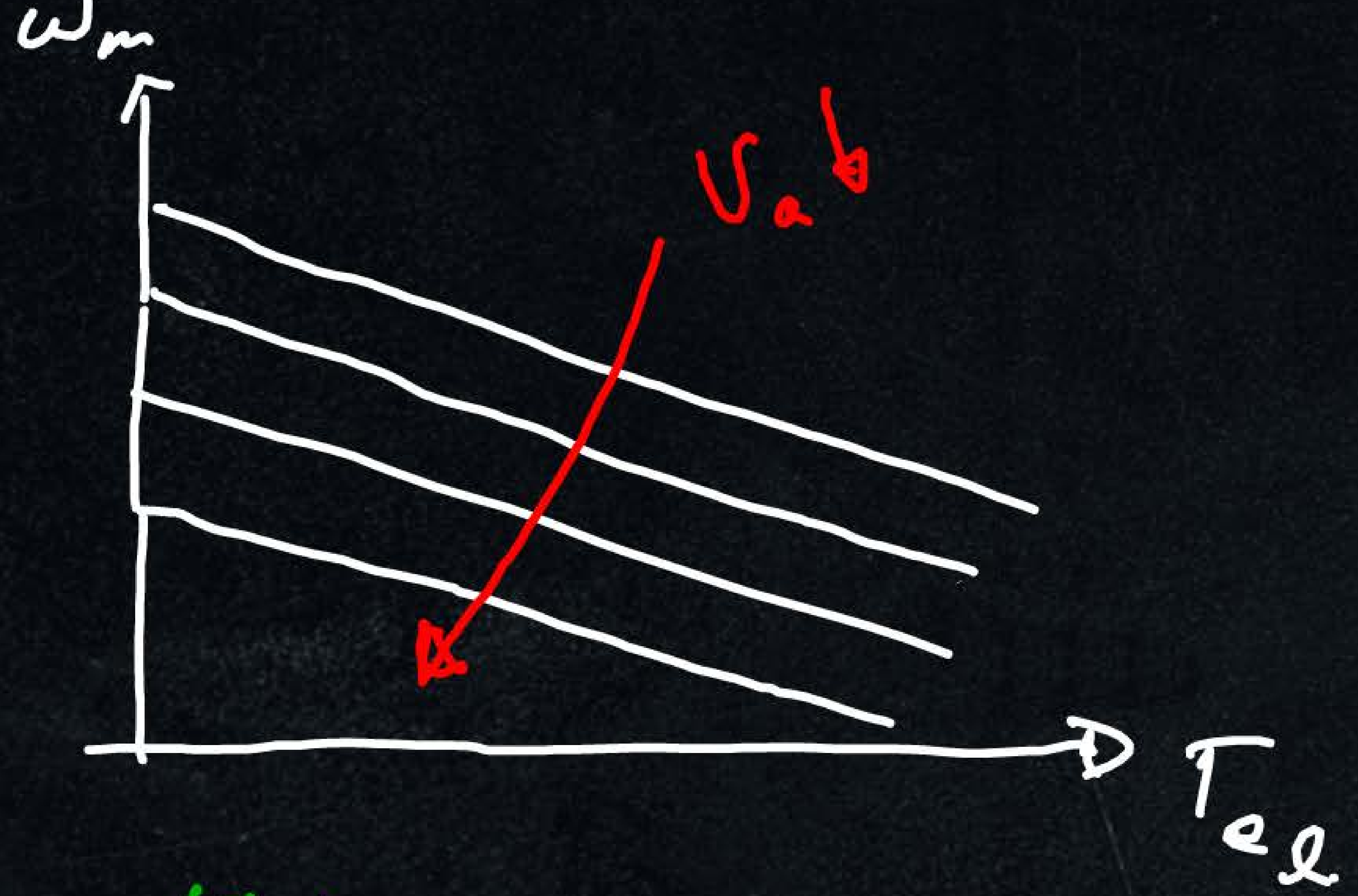
$$V_a = \frac{R_a}{k\phi_f} T_{el} + k\phi_f \omega_m$$

$$\omega_m = \frac{V_a}{k\phi_f} - \frac{R_a}{(k\phi_f)^2} T_{el}$$



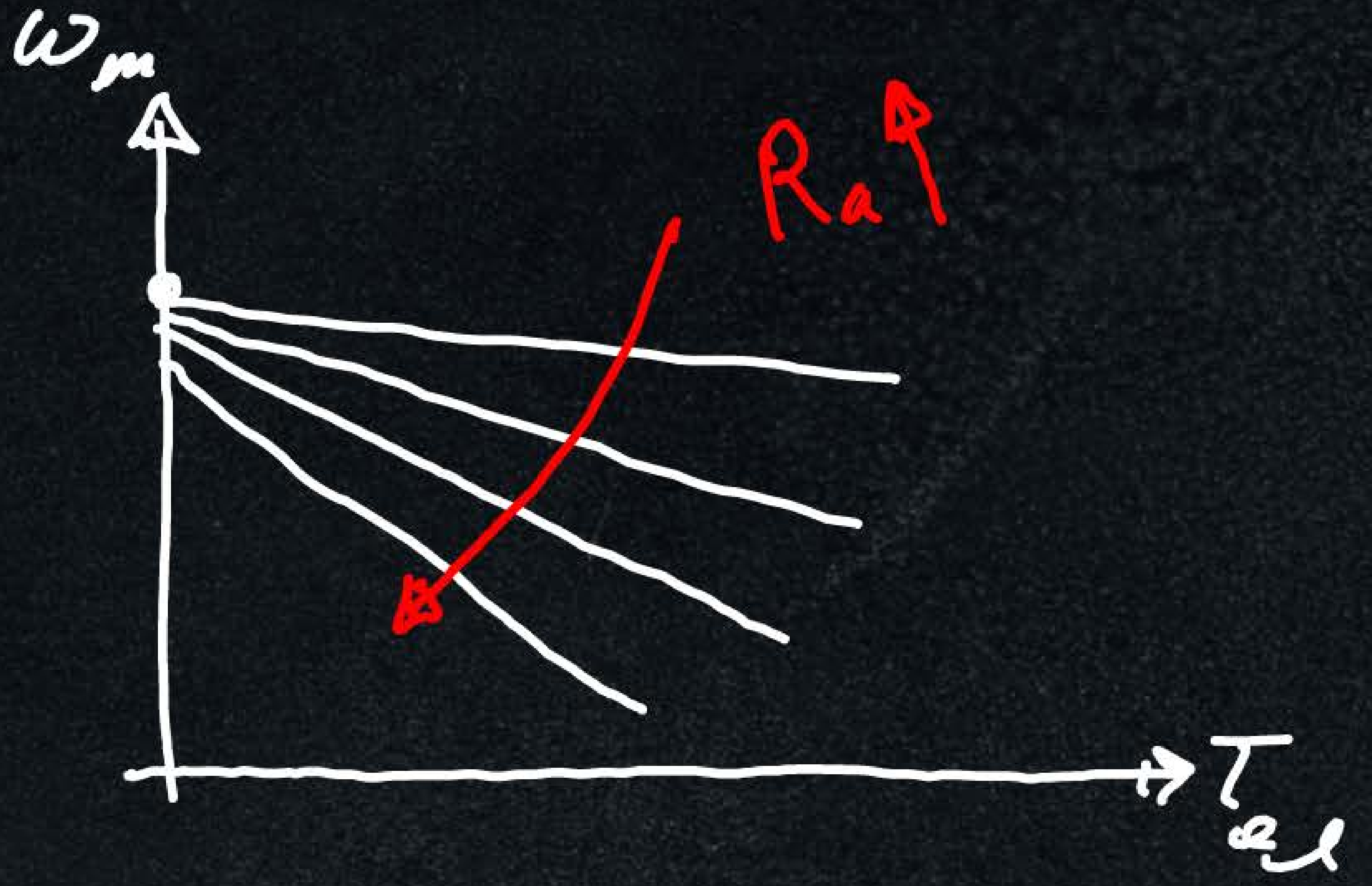
Methods of speed control

1) U_a control



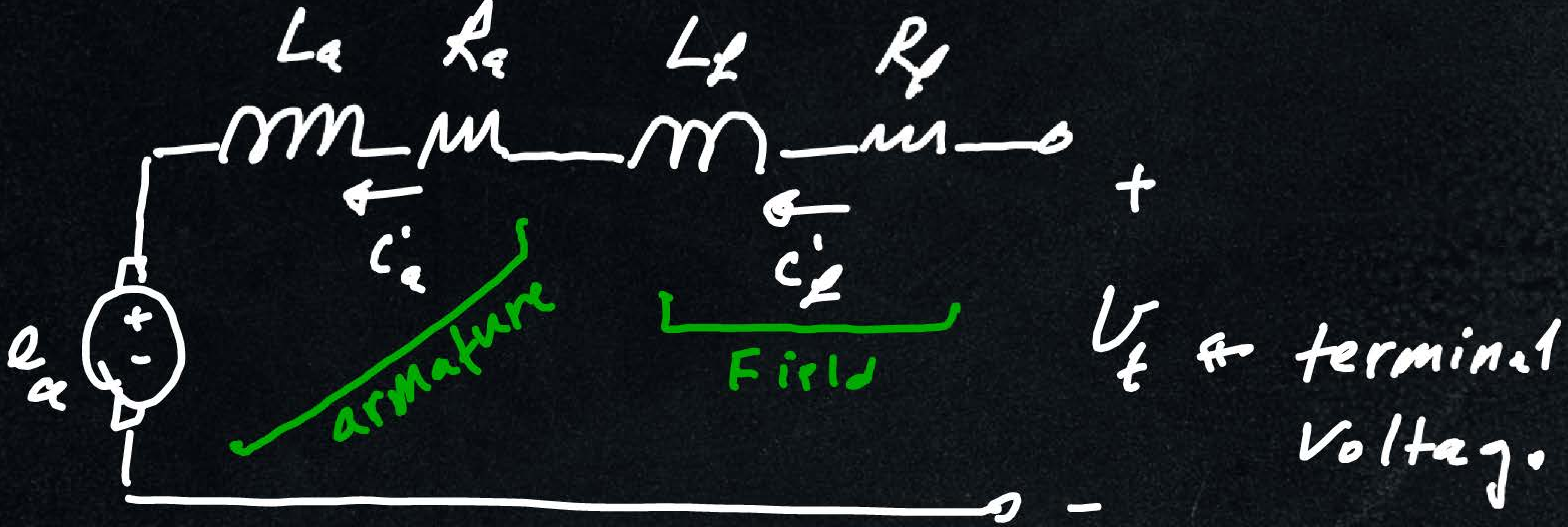
$U_a \downarrow \Rightarrow \omega_m \downarrow$

2) R_a Control



$R_a \uparrow \Rightarrow \omega_m \downarrow$

② Series



$$i_a = i_f$$

$$U_t = R\, i_a + k\Phi_f\, \omega_m$$

where $R = R_f + R_a$, $\Phi_f = c\, i_f$

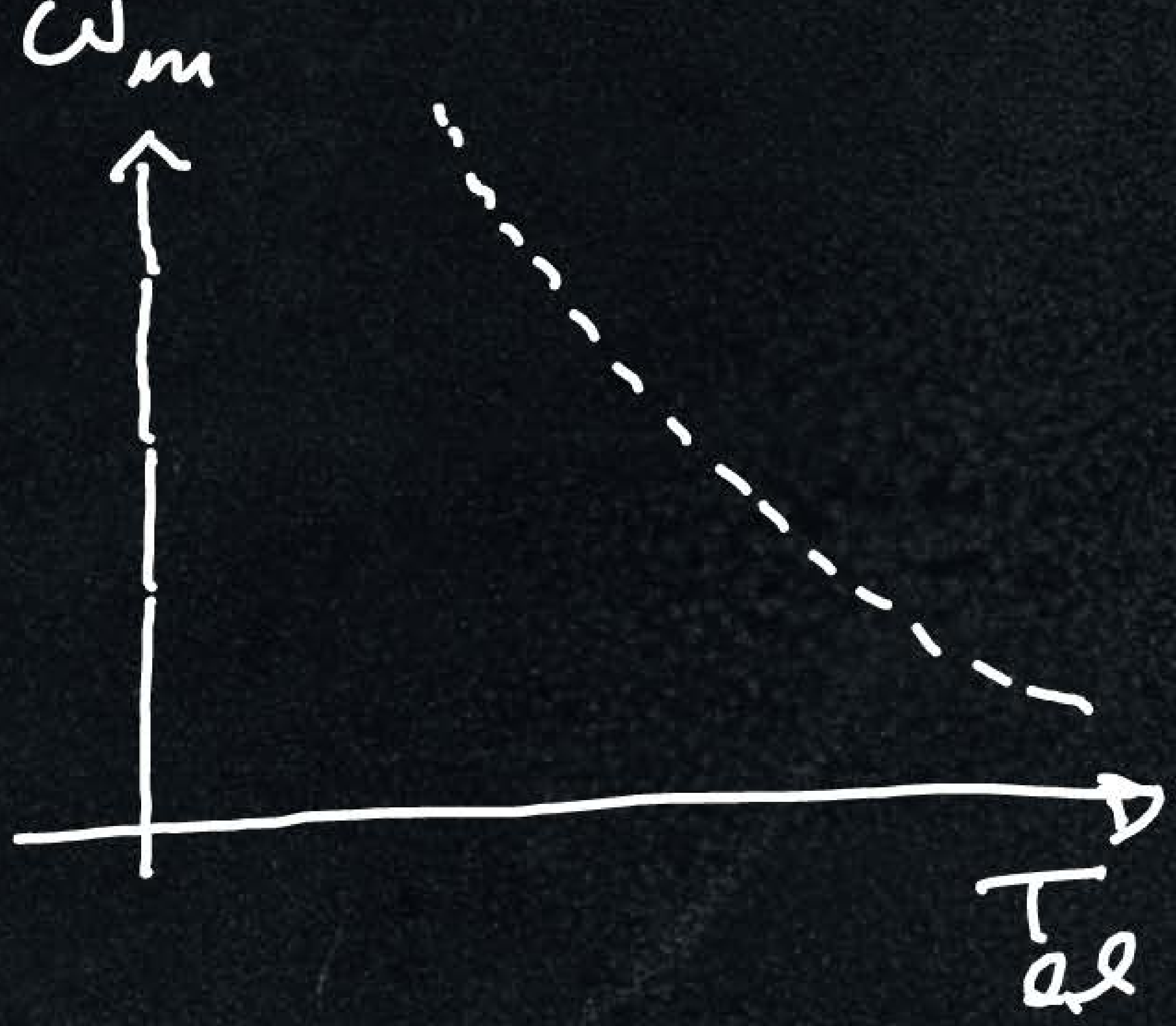
$$U_t = R\, i_a + k c\, i_a \omega_m \quad \text{---} ①$$

$$T_{el} = k\Phi_f\, i_a = k c\, i_a^2$$

$$\Rightarrow i_a = \sqrt{T_{el}} / \sqrt{kc} \quad \text{---} ②$$

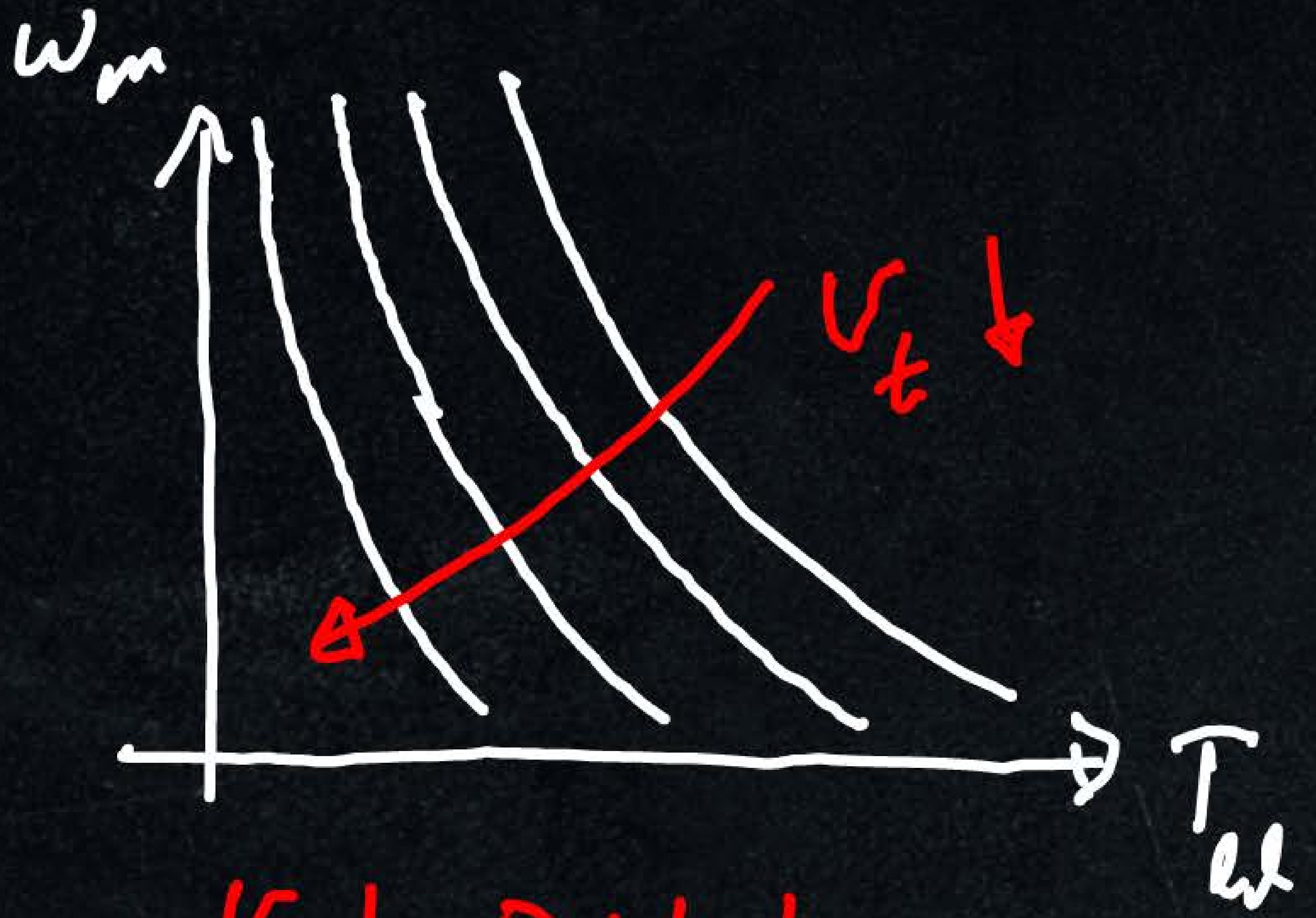
$$② \rightarrow ① \quad U_t = \frac{R}{\sqrt{kc}}\sqrt{T_{el}} + \sqrt{kc}\sqrt{T_{el}}\,\omega_m$$

$$\Rightarrow \quad \omega_m = \frac{U_t}{\sqrt{kc}} \cdot \frac{1}{\sqrt{T_{el}}} - \frac{R}{kc}$$



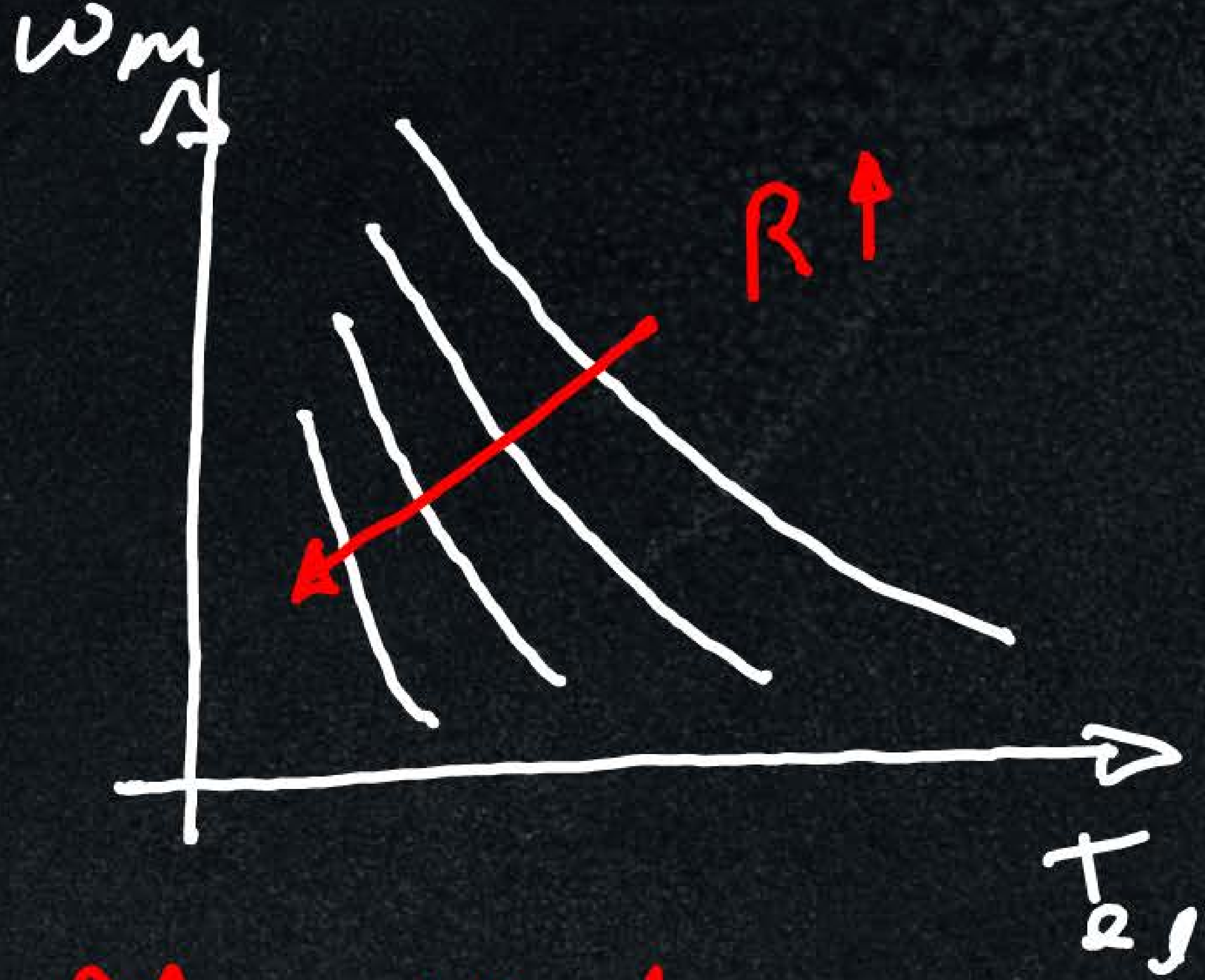
Methods of speed control

1) V_t control



$V_t \downarrow \Rightarrow \omega_m \downarrow$

2) R control



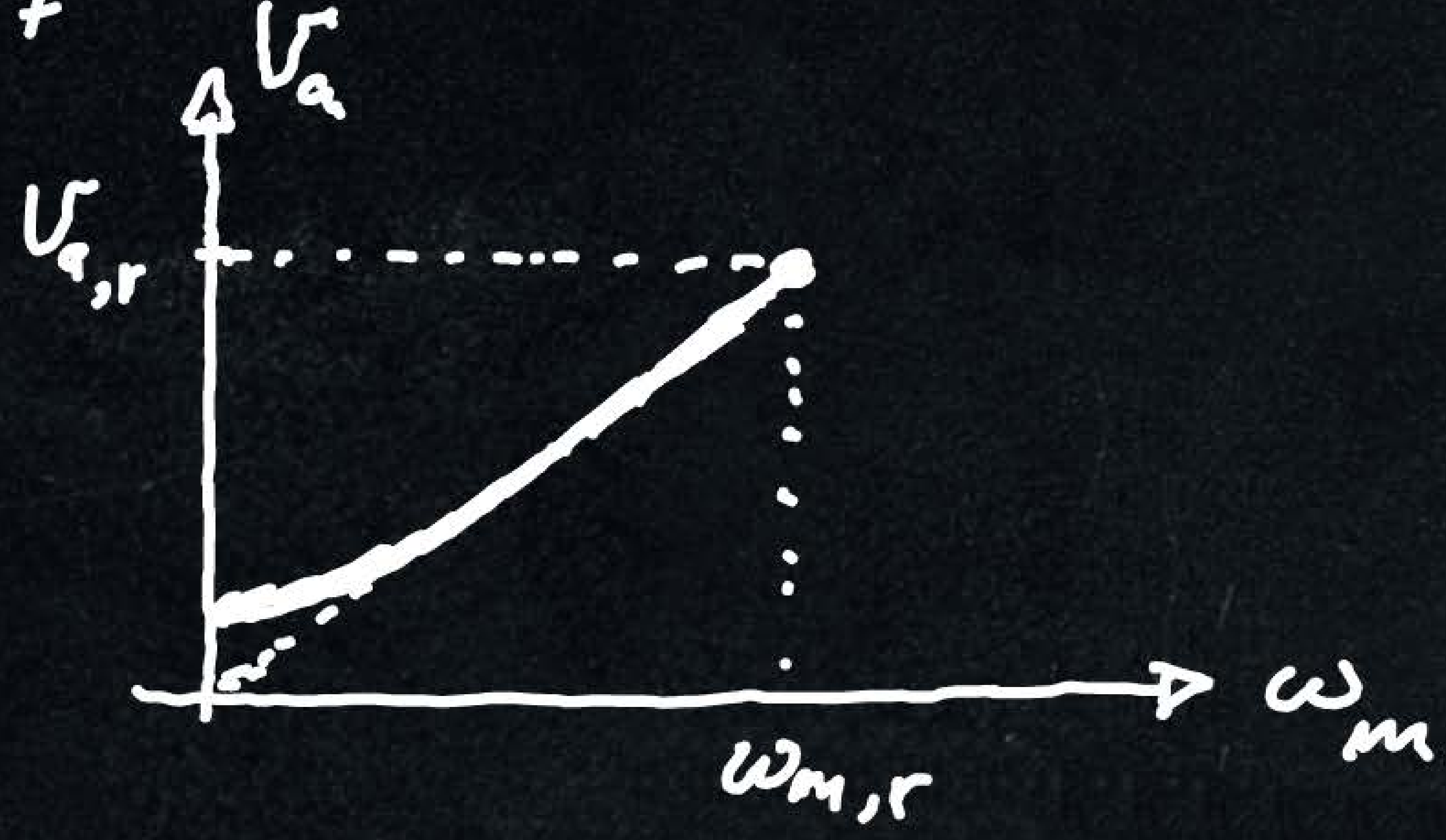
$R \uparrow \rightarrow \omega_m \downarrow$

Principle of DC Machine Drive

$$v_a = R_a i_a + k\Phi_f \omega_m \Rightarrow \boxed{\omega_m = \frac{v_a - R_a i_a}{k\Phi_f} \cong \frac{v_a}{k\Phi_f}}$$

Armature control → Ideal for speeds lower than $\omega_{m,r}$.

$\phi_f = \text{constant} \Rightarrow \omega_m \propto v_a$



$\omega_{m,r}$: Rated speed

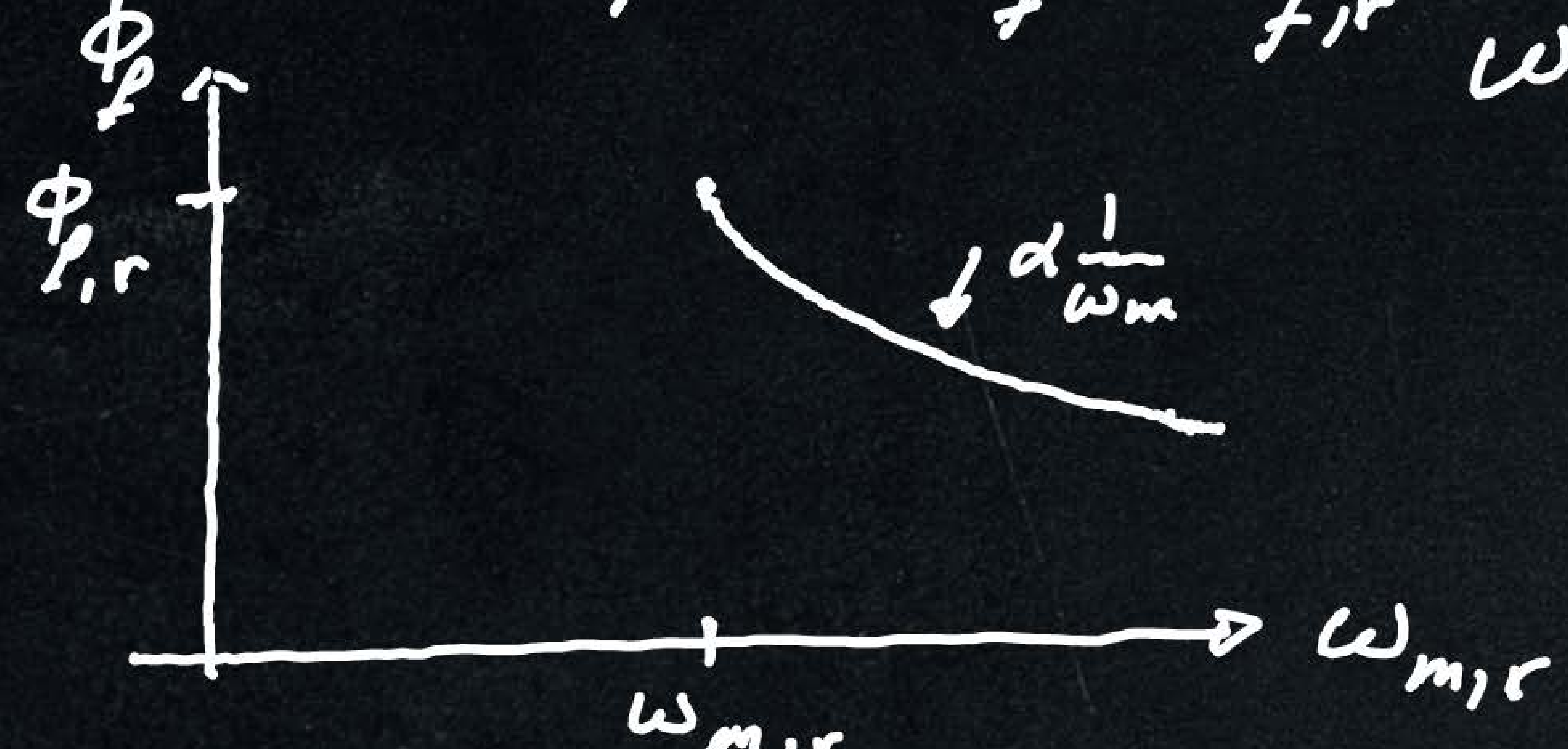
$v_{a,r}$: Rated voltage

Field control

→ Ideal for speeds above $\omega_{m,r}$

$$V_a = V_{a,r} \Rightarrow \omega_m = \frac{V_{a,r}}{K\Phi_f} \Rightarrow \Phi_f \propto \frac{1}{\omega_m}$$

$$\omega_m \uparrow \Rightarrow \Phi_f \downarrow \qquad \Phi_f = \Phi_{f,r}\frac{\omega_{m,r}}{\omega_m}$$



$\Phi_{f,r}$: Rated flux

$\omega_{m,r}$: Rated speed

Armature and field control

$\omega_m \leq \omega_{m,r} \Rightarrow \Phi_f = \Phi_{f,r}$ & $V_a = R_a i_a + k \Phi_f \omega_m$

Assume $i_a = i_{a,r}$ where $i_{a,r}$ is the rated current

$T_{el} = k \Phi_{f,r} i_{a,r} = \text{constant}$

$P_a = T_{el} \omega_m \propto \omega_m$

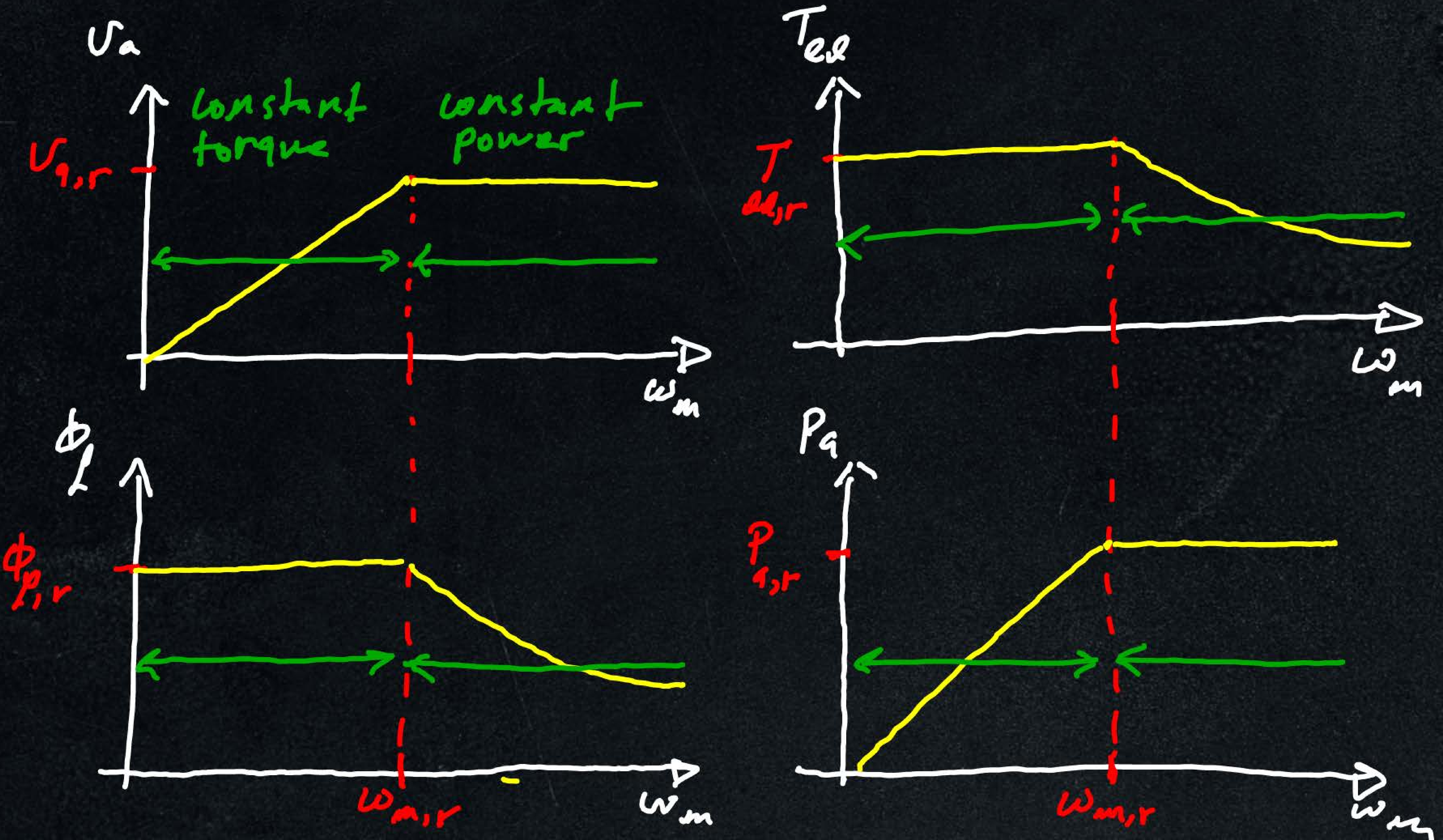
constant torque region

$$\omega_m \geq \omega_{m,r} \Rightarrow \phi_f = \phi_{f,r}\,\frac{\omega_{m,r}}{\omega_m} \quad + \quad V_a = V_{a,r}$$

Assume $i_a' = i_{a,r}'$

$$T_{el} = K\phi_f\, i_a' = K\phi_{f,r}\, i_{a,r}'\,\frac{\omega_{m,r}}{\omega_m} \propto \frac{1}{\omega_m}$$

$$P_a = T_{el}\,\omega_m = \underbrace{K\phi_{f,r}\, i_{a,r}'\,\omega_{m,r} = \text{constant}}_{\text{constant power region}}$$



U_a
constant torque
constant power
$U_{a,r}$
ω_m
T_{el}
$T_{el,r}$
ω_m
Φ_f
$\Phi_{f,r}$
$\omega_{m,r}$
ω_m
P_a
$P_{a,r}$
$\omega_{m,r}$
ω_m

Braking and Four-Quadrant operation

Braking

The machine is made to work as a generator producing a negative torque

$$T_{el} - T_L = J_{tot} \frac{d\omega_m}{dt}$$

$\omega_m > 0$ + $(T_{el} - T_L) < 0 \Rightarrow$ mahine acts as a generator

$$T_{el} - T_L = J_{tot}\frac{d\omega_m}{dt} \Rightarrow \Delta t = \frac{J_{tot}}{T_{el} - T_L}\,\Delta\omega_m$$

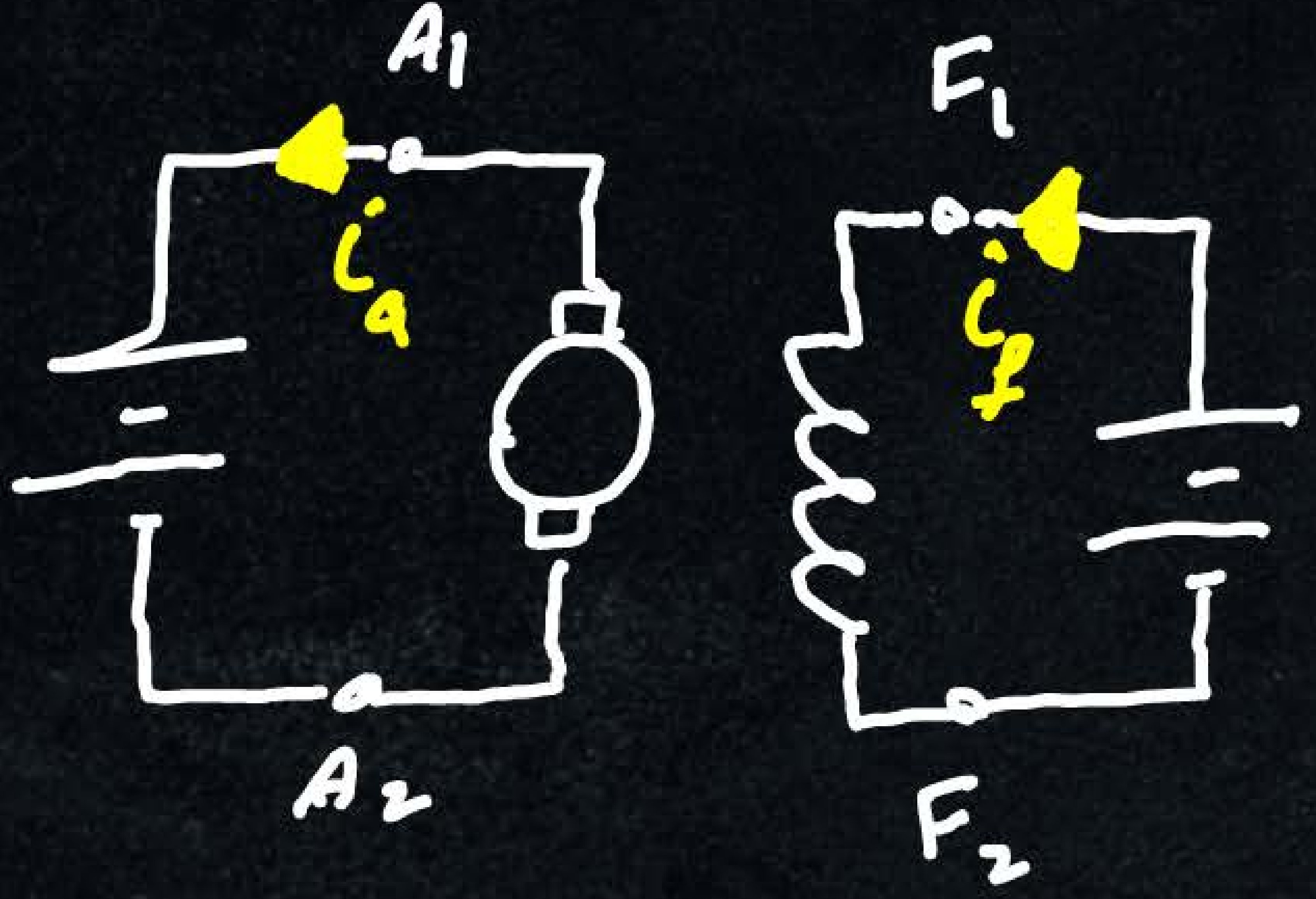
Δt : Stopping time

why do we need braking ?

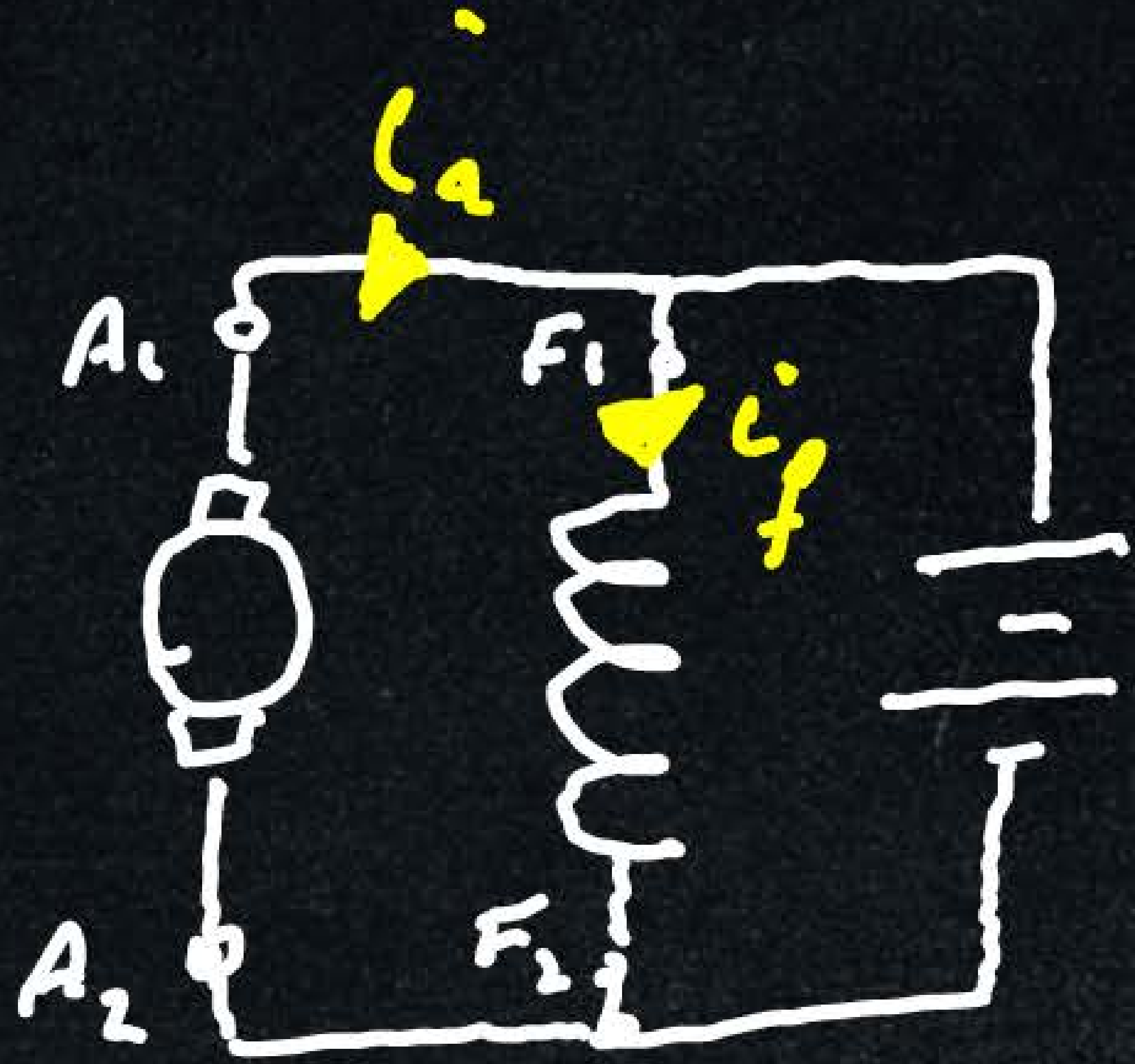
1) Reducing Δt
2) Achieving quick and smooth stops
3) Achieving accurate stops
4) Holding the speed within safe limit

Braking methods

① Regenerative braking → Generated electrical power is usefully employed.

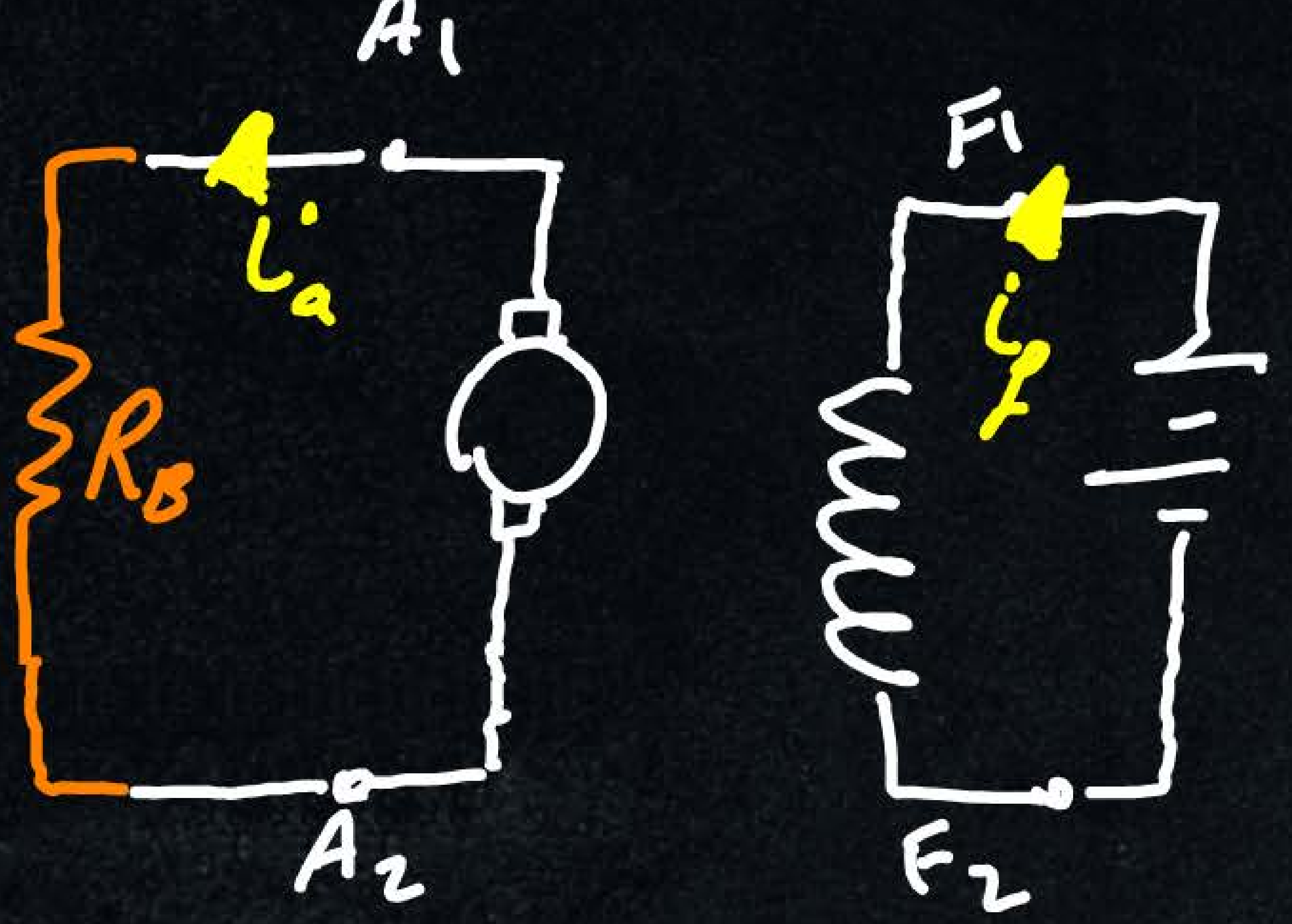


separately

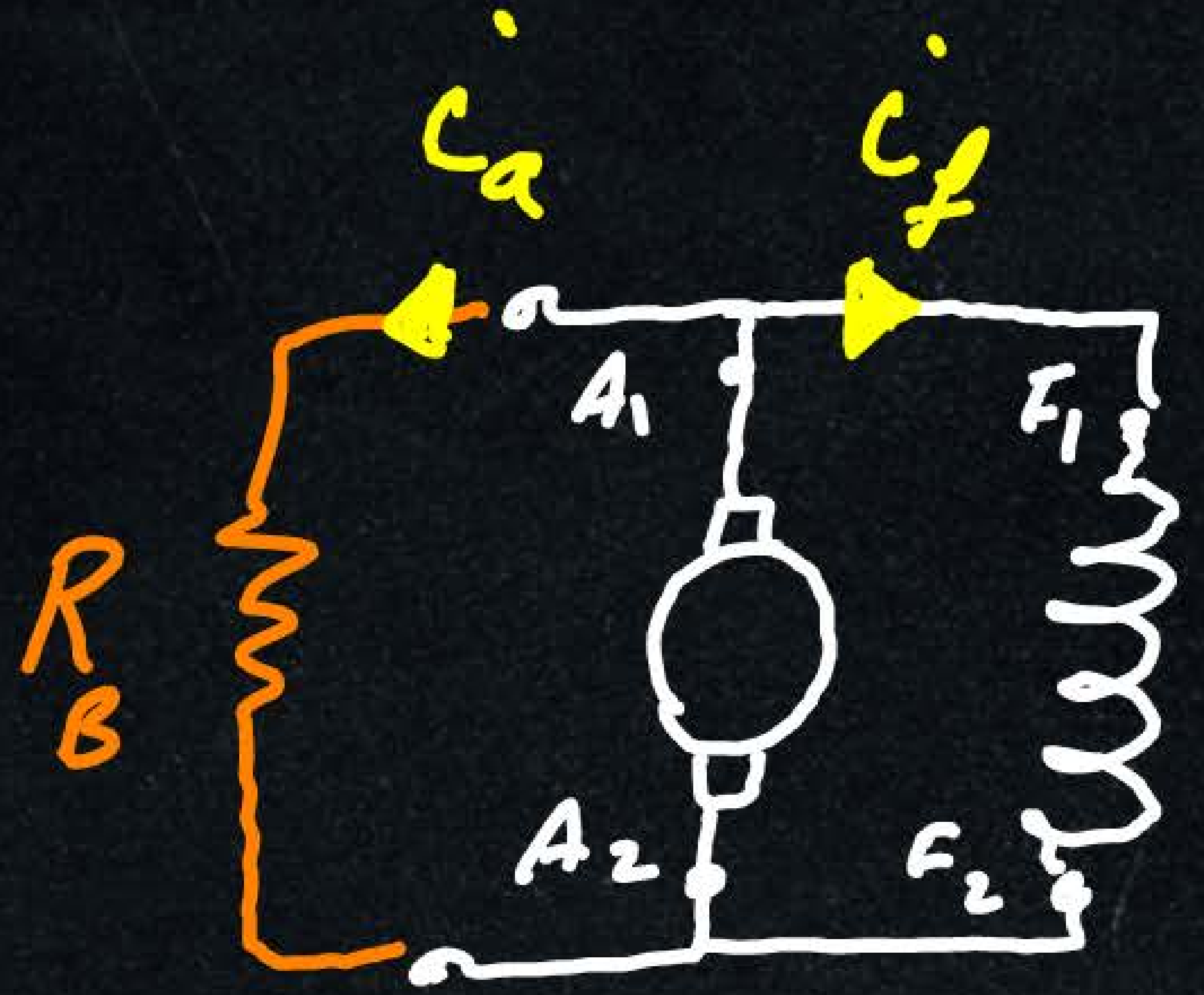


shunt.

② Dynamic braking → It is an inefficient method of braking.

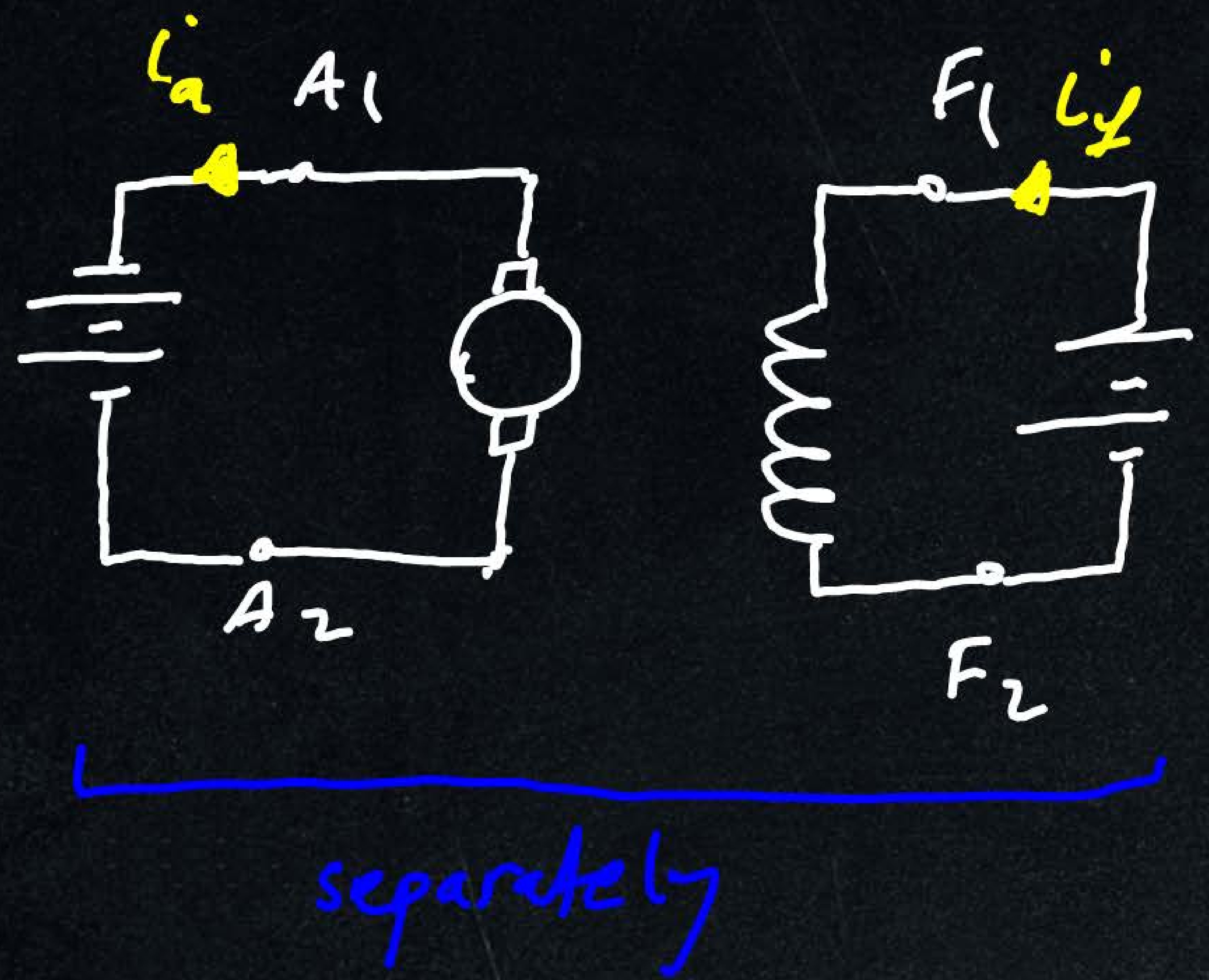


separately



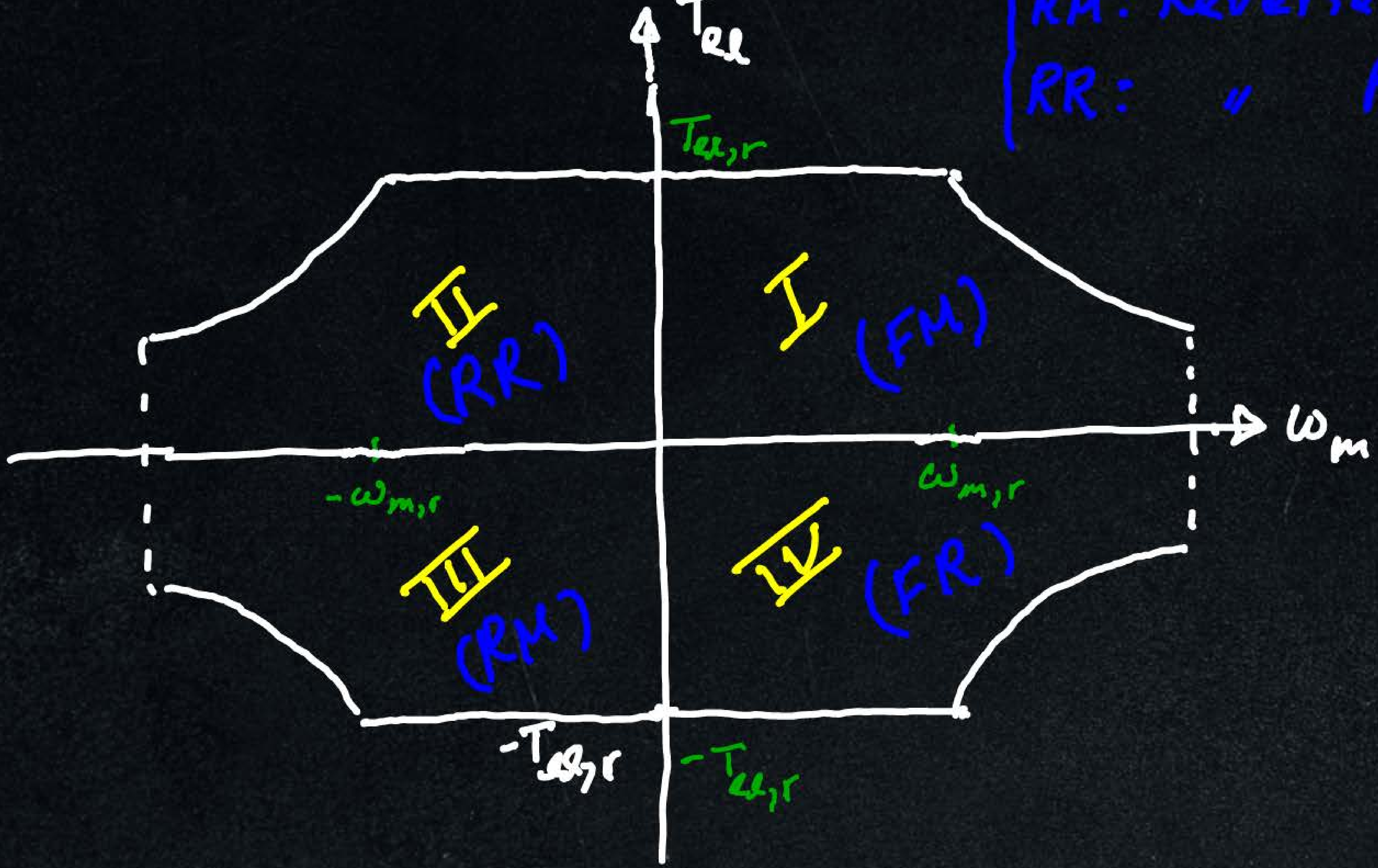
shunt

③ plugging braking → It is a highly inefficient method.



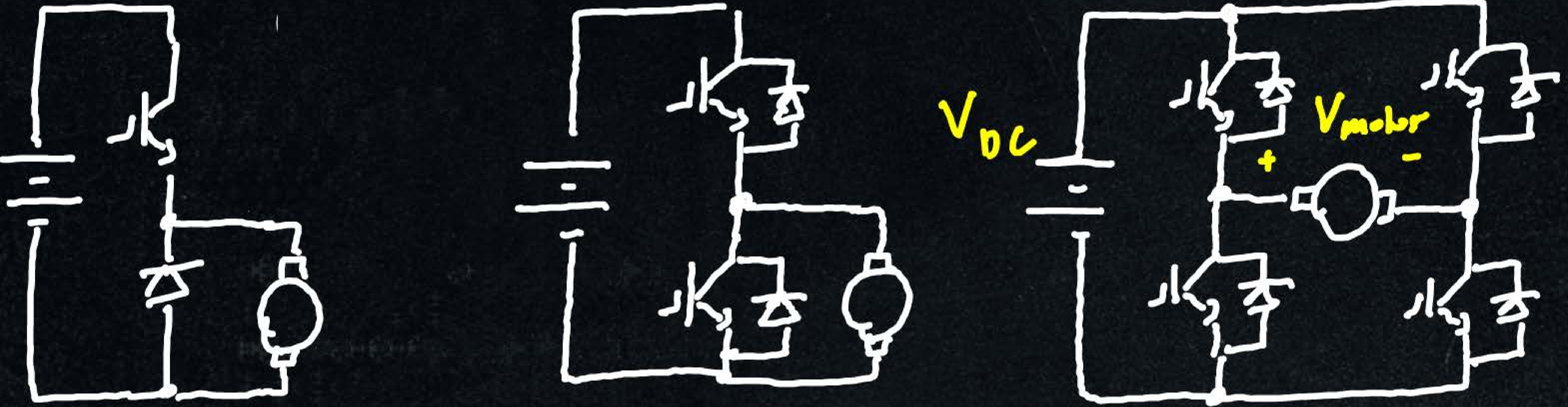
Four-Quadrant operation

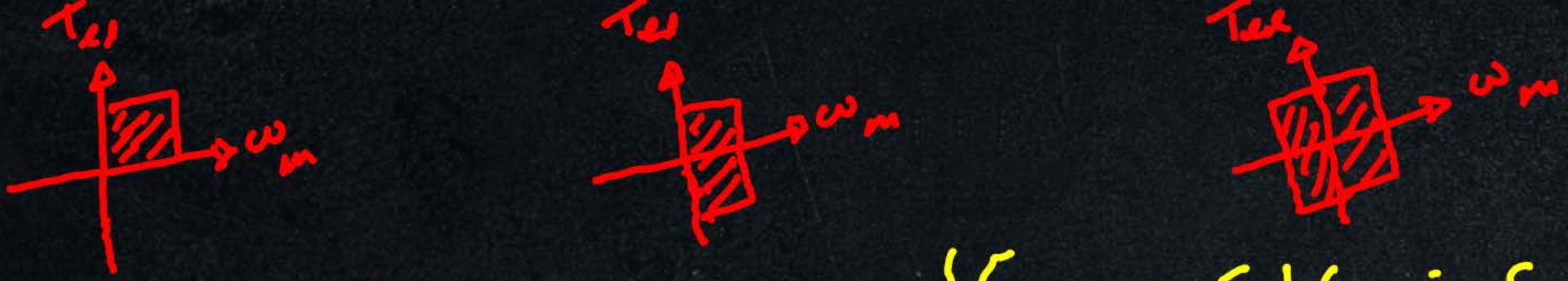
FM: Forward Motoring
FR: " Regeneration
RM: Reverse Motoring
RR: " Regeneration



Function	Quadrant	ω_m	T_{el}	V_a	i_a	P_a
FM	I	+	+	+	+	+
FR	IV	+	−	+	−	−
RM	III	−	−	−	−	+
RR	II	−	+	−	+	−

Chopper Controlled DC Motor Drive

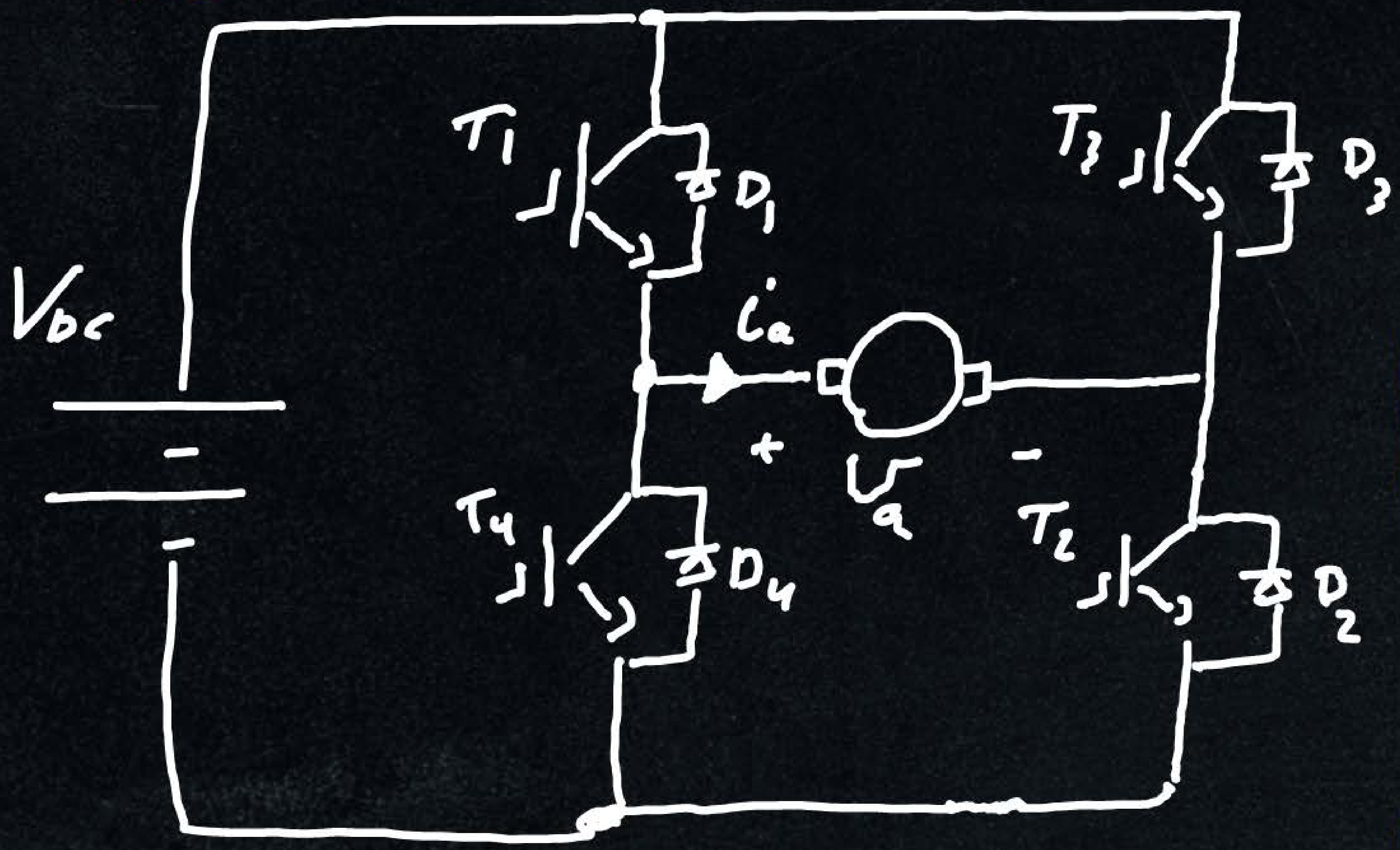


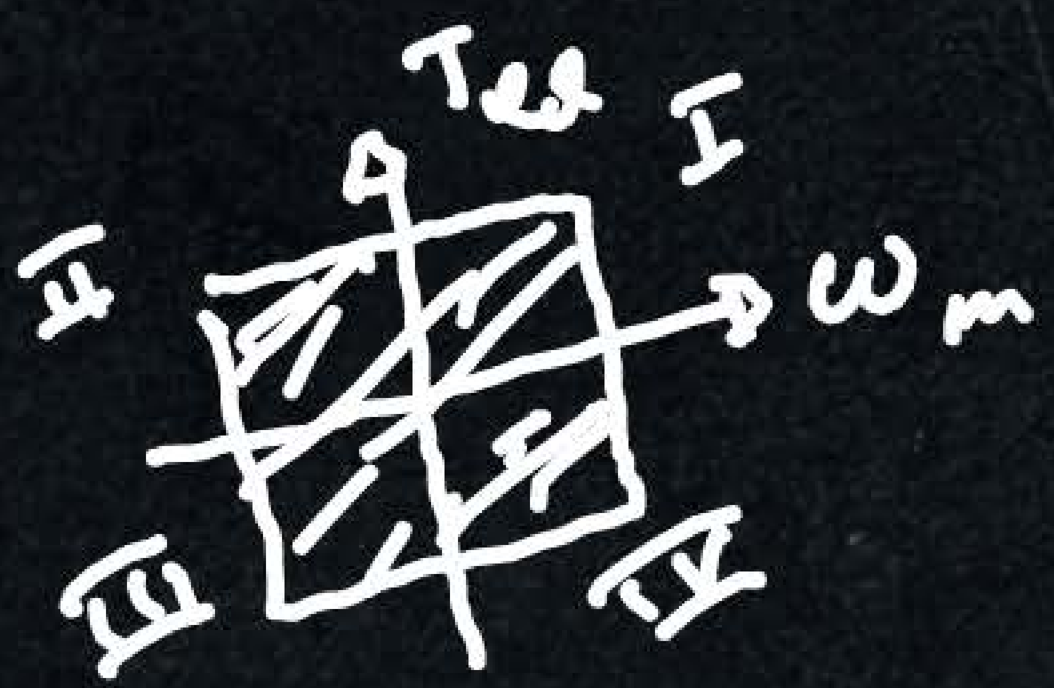


$$0 \leq \delta \leq 1$$

$$V_{motor} = \delta V_{DC} \; ; \; \delta : \text{Duty cycle}$$

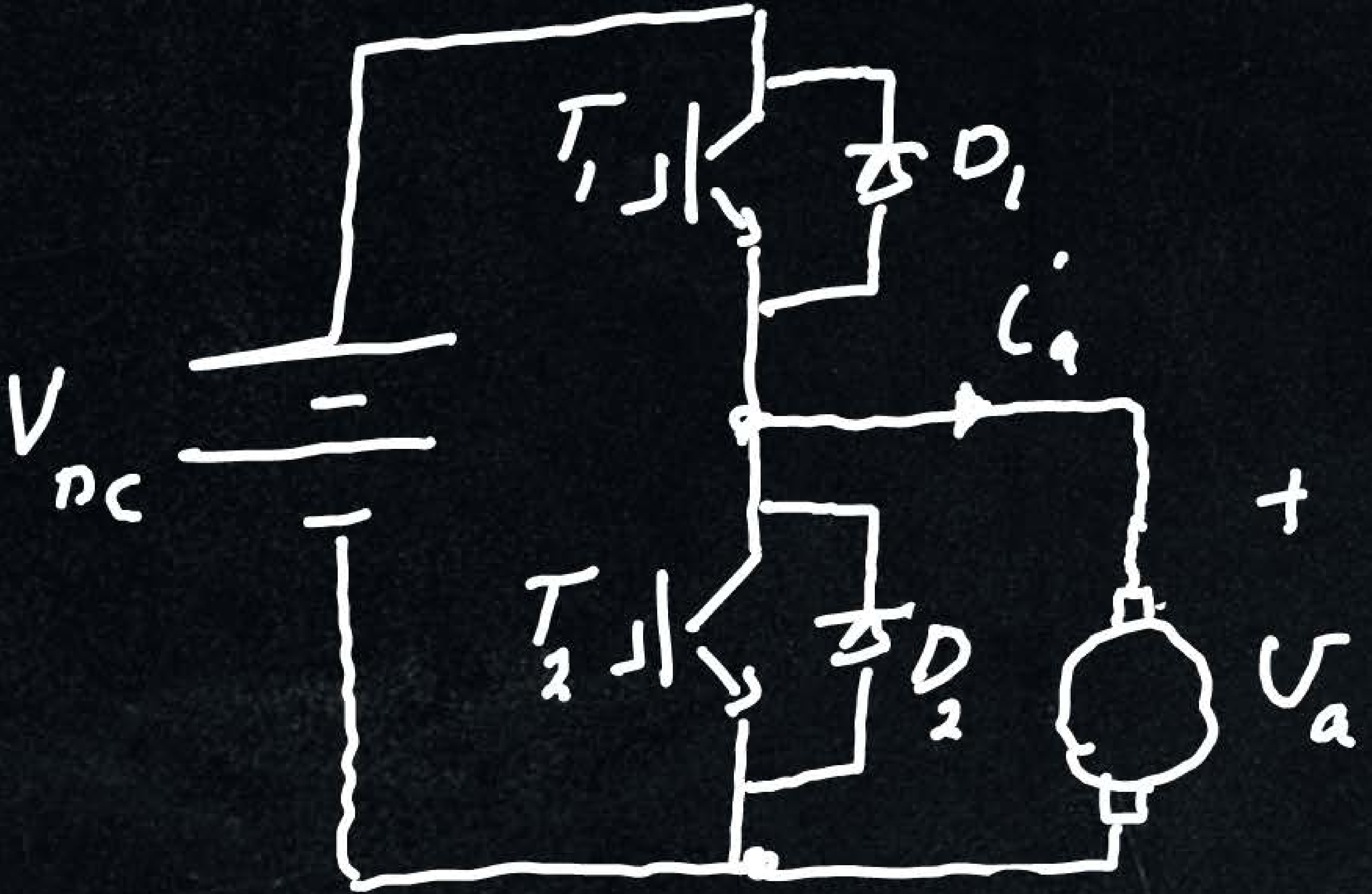
4 quadrant chopper

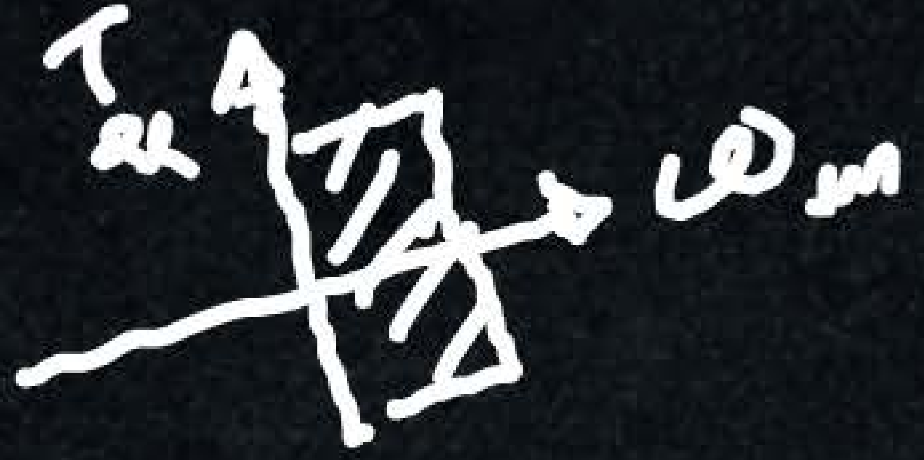




	Conducting Devices	V_a	i_a	V_a =
I	$T_1 T_2$	+	+	V_{DC}
I	$T_1 D_3$ or $T_2 D_4$	0	+	0
II	$D_3 D_4$	−	+	$-V_{DC}$
II	$T_1 D_3$ or $T_2 D_4$	0	+	0
III	$T_3 T_4$	−	−	$-V_{DC}$
III	$T_3 D_1$ or $T_4 D_2$	0	−	0
IV	$D_1 D_2$	+	−	V_{DC}
IV	$T_3 D_1$ or $T_4 D_2$	0	−	0

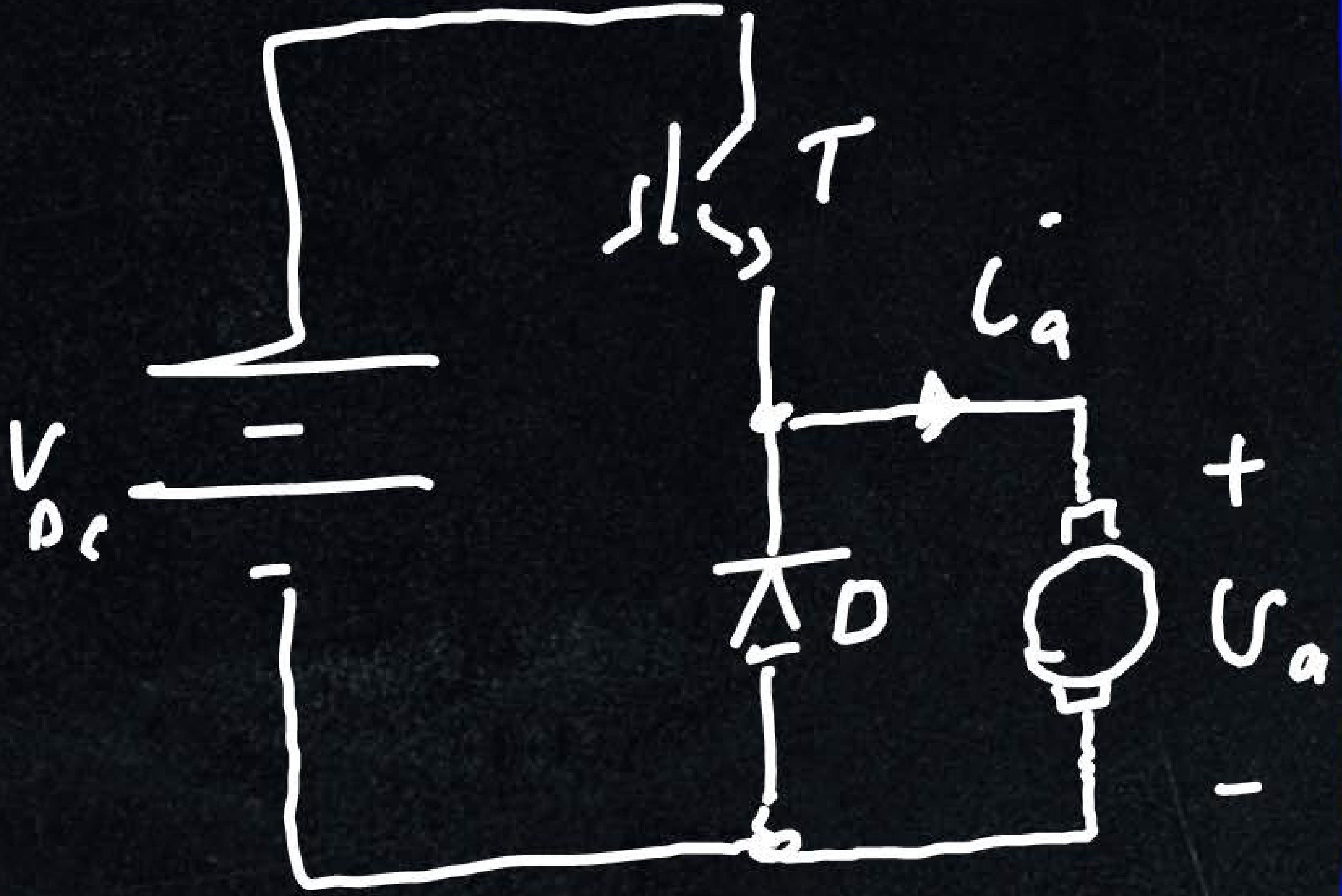
2 quadrant chopper

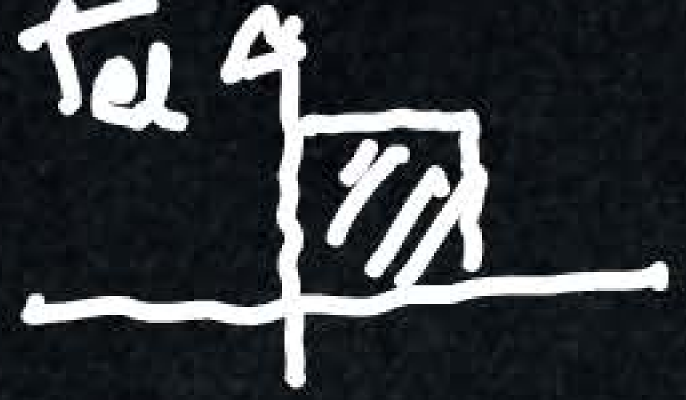




Q	Conducting Devices	V_a	i_a	V_a
I	T_1	+	+	V_{DC}
I	D_2	0	+	0
IV	T_2	0	−	0
IV	D_1	+	−	V_{DC}

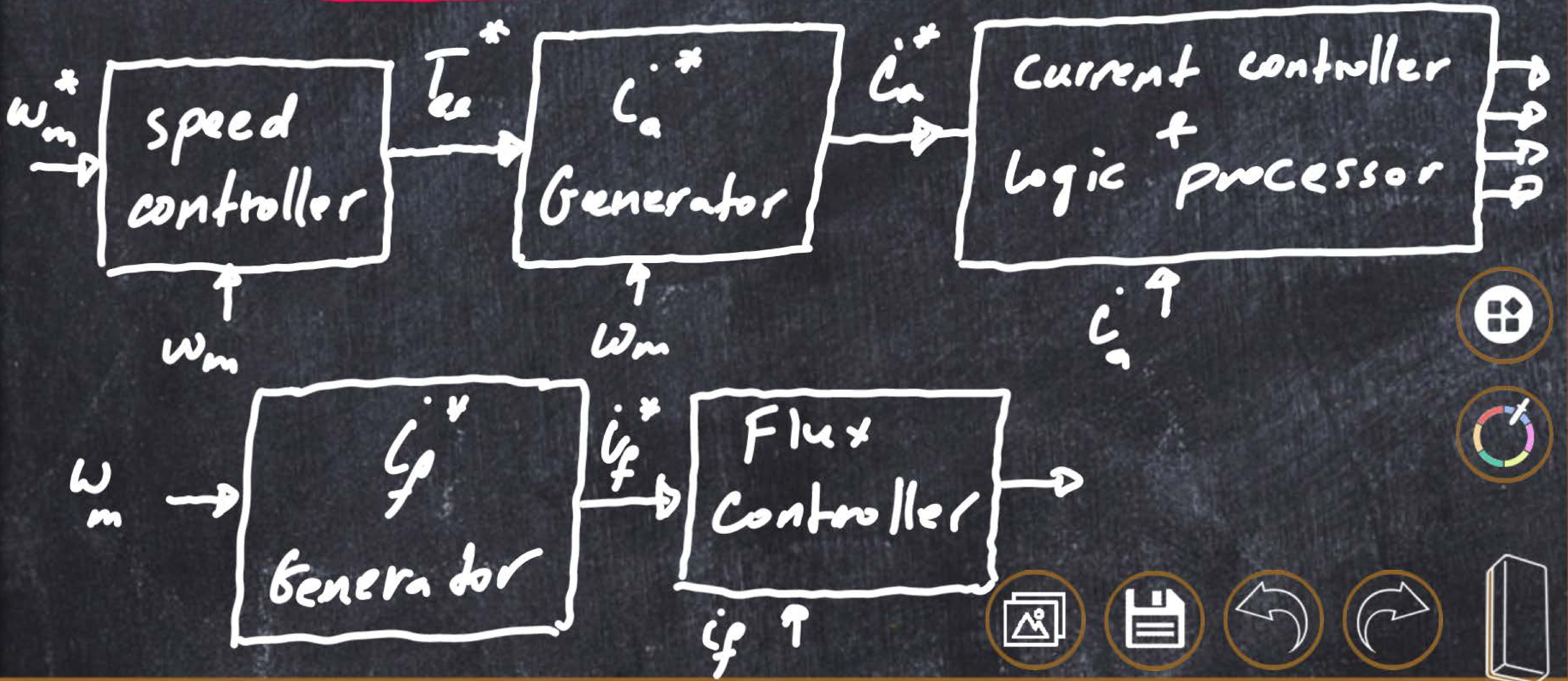
1 quadrant chopper



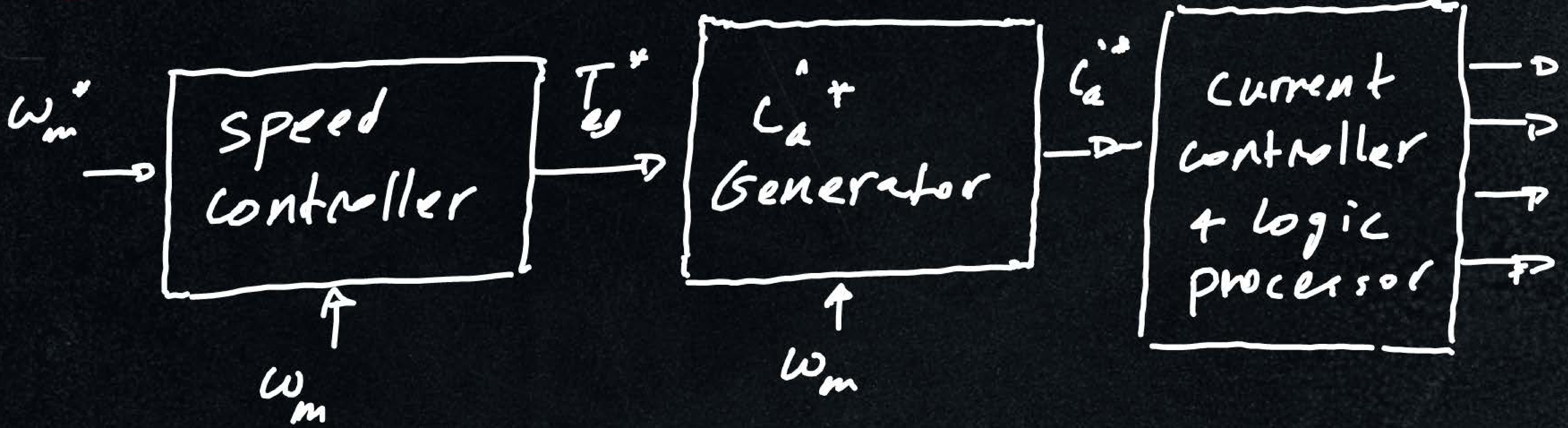


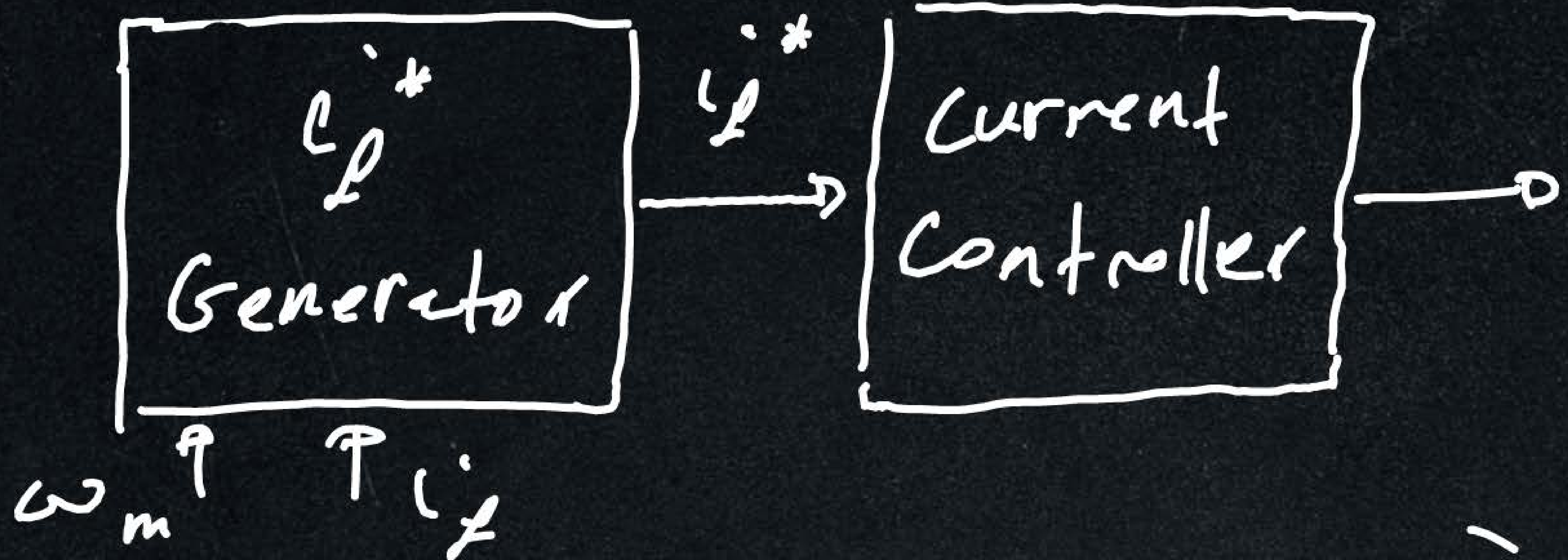
Q	conducting Devices	v_a	i_a	v_a
I	T	+	+	V_{DC}
I	D	0	+	0

Block Diagram of control system



Block Diagram of Control System





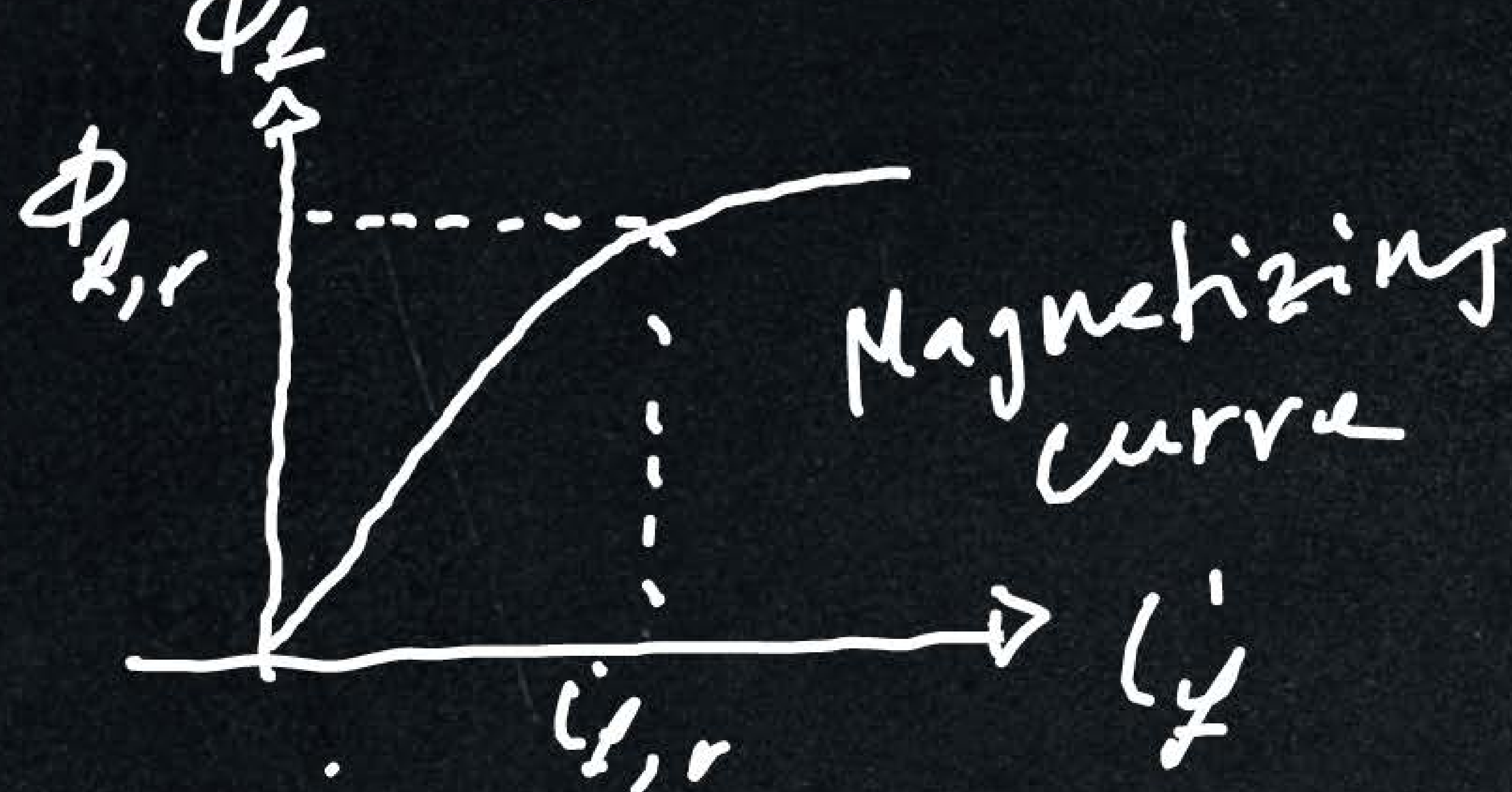
i_f^* Generator

If $\omega_m \leq \omega_{m,r}$

$$i_f^* = i_{f,r} = \frac{\Phi_{f,r}}{C}$$

If $\omega_m \geq \omega_{m,r}$

$$i_f^* = i_{f,r} \frac{\omega_{m,r}}{\omega_m}$$



$i_a'^*$ Generator

$$T_{el}^* = K\Phi_f\, i_a'^* \Longrightarrow i_a'^* = \frac{T_{el}^*}{K\Phi_f}$$

If $\omega_m \leq \omega_{m,r}$

$$i_a'^* = \frac{1}{K\Phi_{f,r}}\, T_{el}^*$$

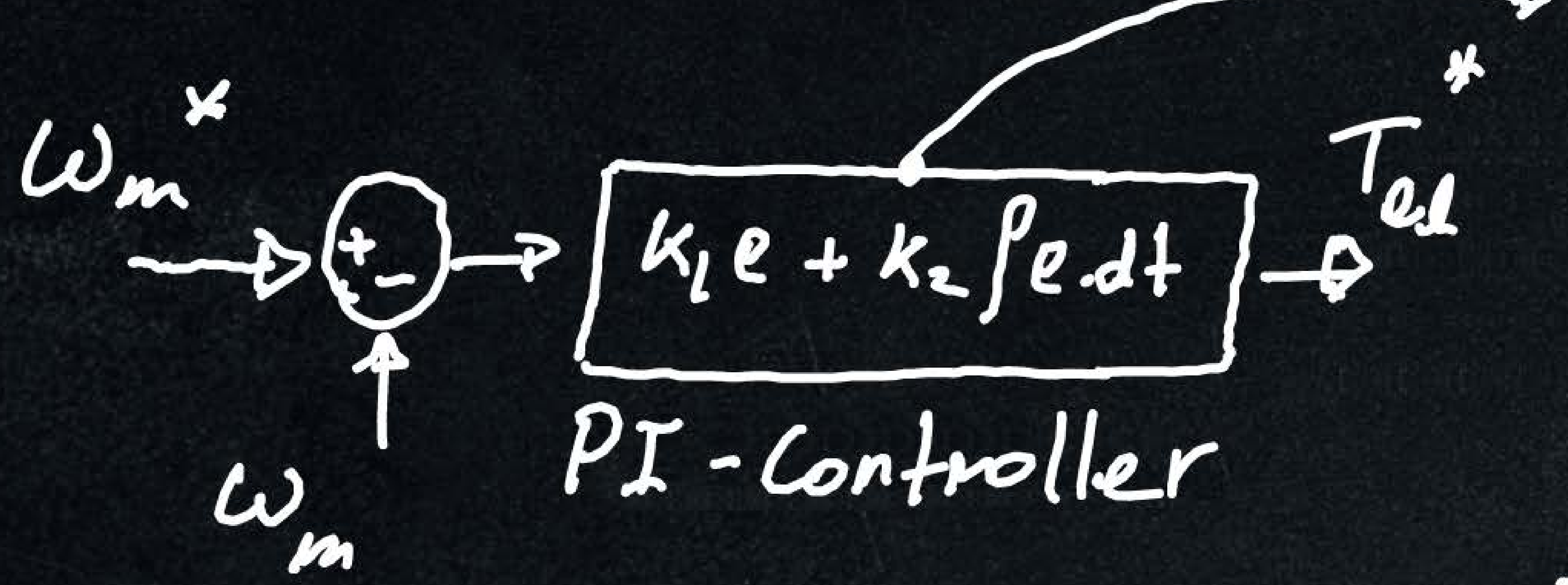
If $\omega_m \geq \omega_{m,r}$

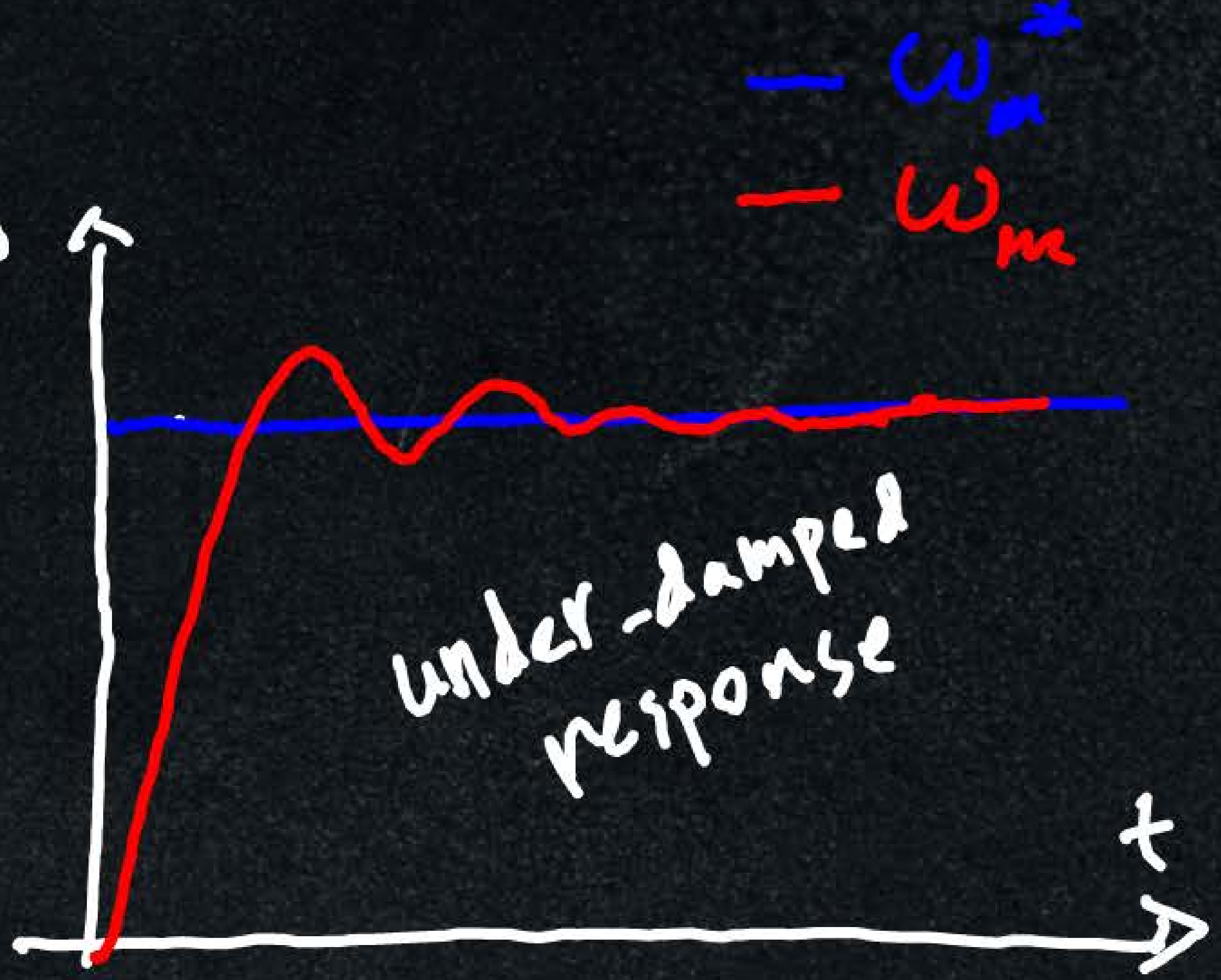
$$i_a'^* = \frac{\omega_m}{K\Phi_{f,r}\,\omega_{m,r}} \cdot T_{el}^*$$

Speed Controller

It is designed using Newton's 2^{nd} law:

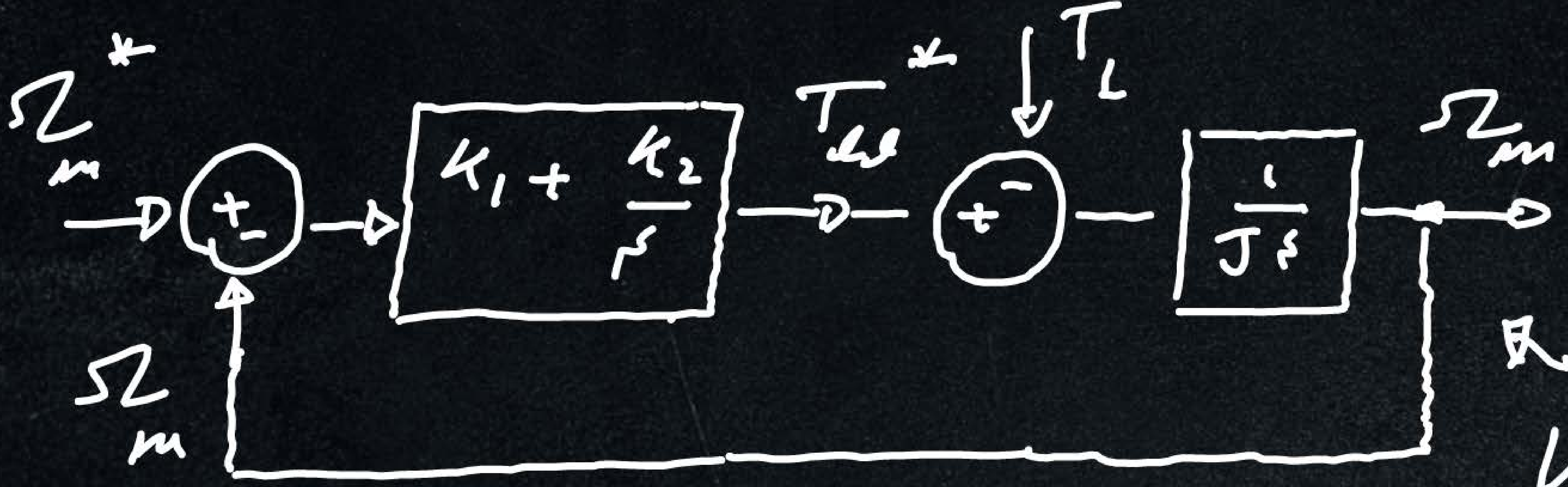
$$T_{el} = T_L + J \frac{d\omega_m}{dt}$$





Design of speed controller in s-domain

$$T_{el} = T_L + J\frac{d\omega_m}{dt} \xrightarrow{\mathcal{L}} T_{el} = T_L + sJ\Omega_m$$



Closed loop control system

Transfer Function

$$\Omega_m = T_1 \Omega_m^* + T_2 T_L$$

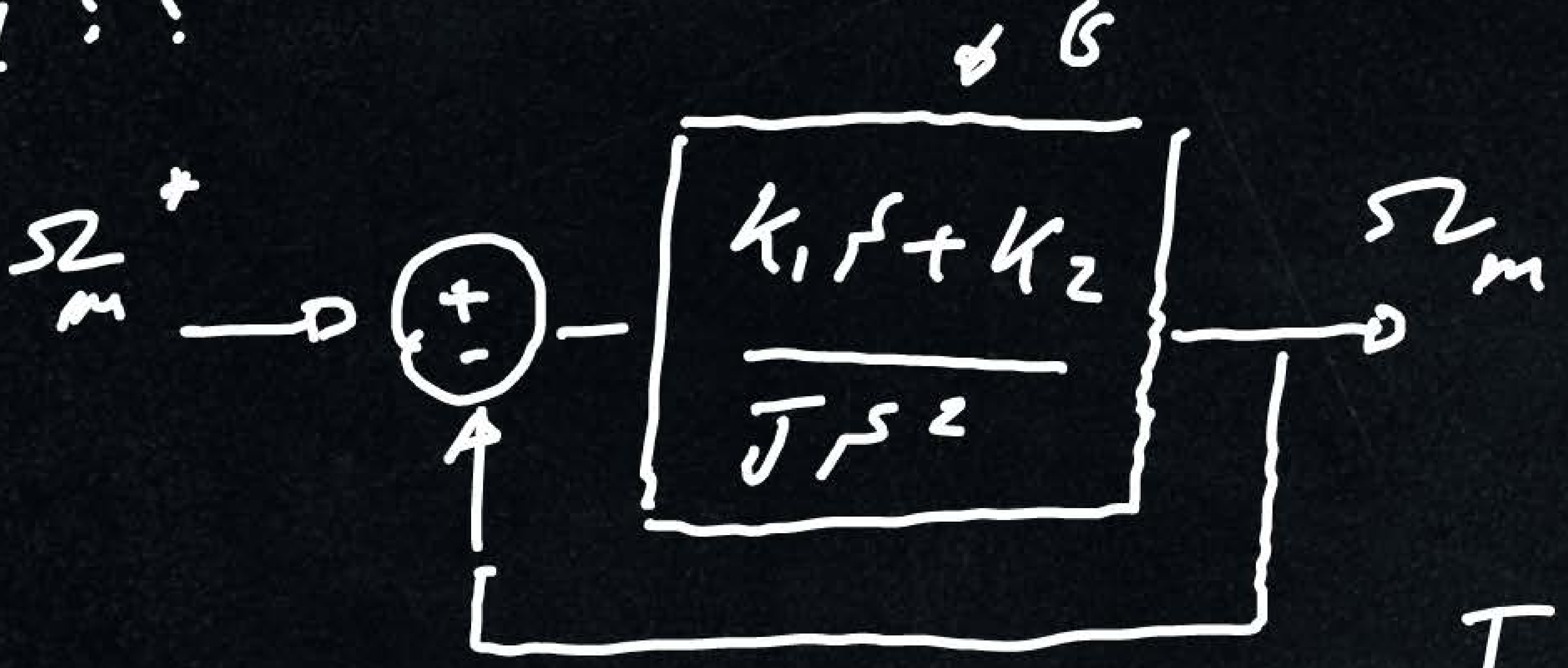
$$T_1 = \frac{\Omega_m}{\Omega_m^*} \quad \text{when} \quad T_L = 0$$

$$T_2 = \frac{\Omega_m}{T_L} \quad \text{when} \quad \Omega_m^* = 0$$

superposition

"Linear system"

T_1 ??



$$T_1 = \frac{G}{1+G}$$

$$T_1 = \frac{(K_1 s + K_2)/(J s^2)}{1 + \left[(K_1 s + K_2)/(J s^2)\right]}$$

$$T_1 = \frac{K_1 s + K_2}{J s^2 + K_1 s + K_2} = \frac{K_1}{J}\left[\frac{s + K_2/K_1}{s^2 + \left(\frac{K_1}{J}\right) s + \left(\frac{K_2}{J}\right)}\right]$$

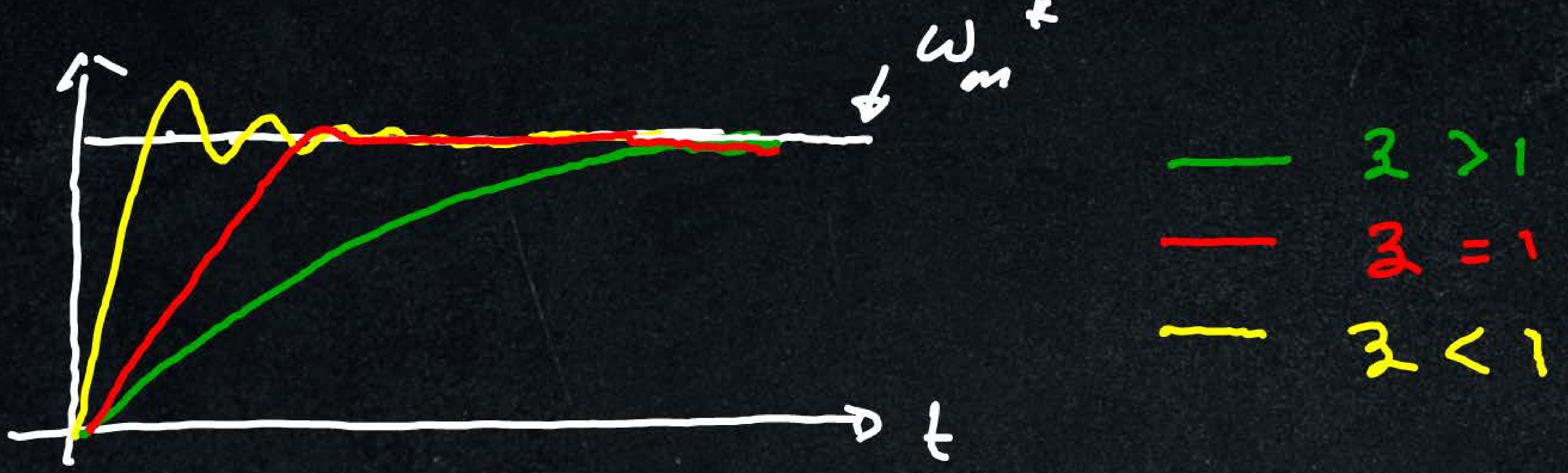
In general $T_1 = K \dfrac{s + a}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

ζ : Damping ration , ω_n : Natural frequency

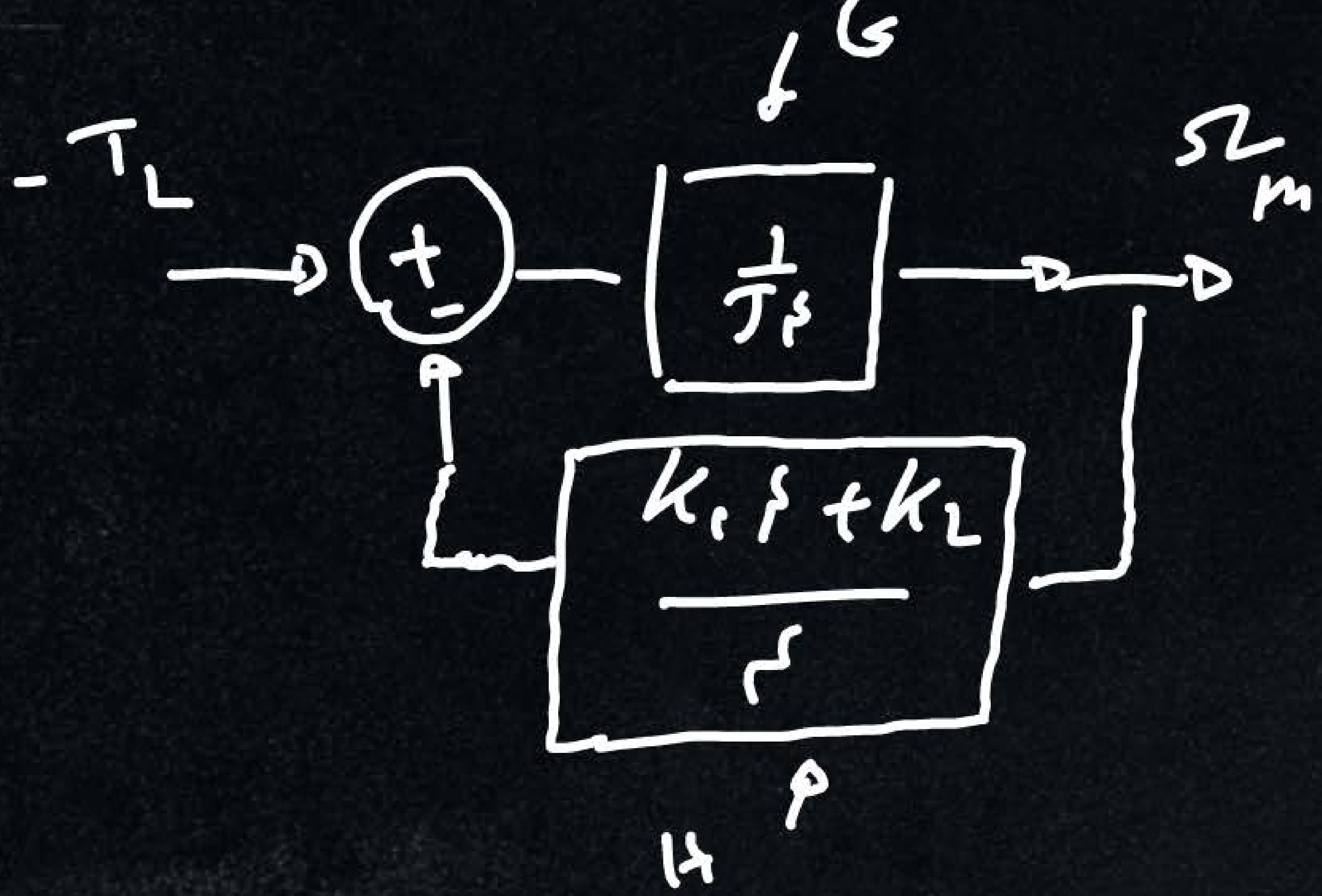
$\zeta < 1$ underdamped

$\zeta = 1$ critically damped

$\zeta > 1$ overdamped .



T_2 ??



$$T_2 = \frac{-G}{1 + GH}$$

$$T_2 = \frac{-1/Js}{1 + \frac{K_1 s + K_2}{Js^2}}$$

$$T_2 = \frac{-s}{Js^2 + K_1 s + K_2} = \frac{-1}{J}\left[\frac{s}{s^2 + \left(\frac{K_1}{J}\right)s + \left(\frac{K_2}{J}\right)}\right]$$

$$\Omega_m = T_1 \Omega_m^* + T_2 T_L$$

$$T_1 = \frac{k_1}{J}\left[\frac{s + \frac{k_2}{k_1}}{s^2 + \frac{k_1}{J}s + \frac{k_2}{J}}\right], \quad T_2 = \frac{-1}{J}\left[\frac{s}{s^2 + \frac{k_1}{J}s + \frac{k_2}{J}}\right]$$

$$w_{m,ss} = \lim_{t\to\infty} \omega_m(t) = \lim_{s\to 0} s\,\Omega_m = \lim_{s\to 0} s T_1 \Omega_m^* + \lim_{s\to 0} s T_2 T_L$$

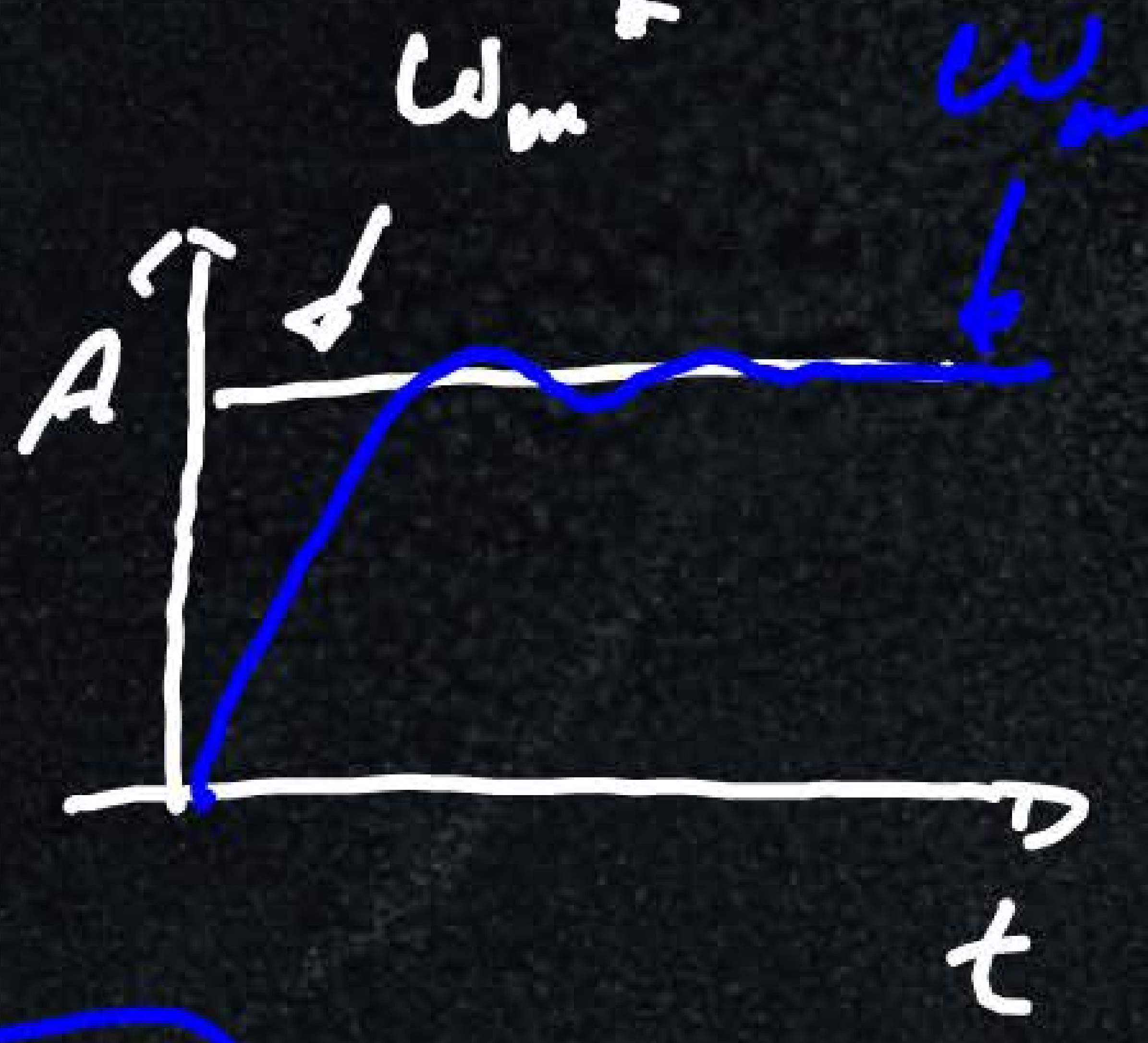
Assume $\omega_m^* = A\,u(t)$ & $T_L = B\,u(t)$

$$\xrightarrow{\mathcal{L}} \Omega_m^* = A/s \quad \& \quad T_L = B/s$$

$$\omega_{m,ss} = \lim_{s\to 0} \cancel{s}\, T_1 \frac{A}{\cancel{s}} + \lim_{s\to 0} \cancel{s}\, T_2 \frac{B}{\cancel{s}}$$

$$\omega_{m,ss} = A \overbrace{\lim_{s\to 0} T_1}^{1} + B \overbrace{\lim_{s\to 0} T_2}^{0}$$

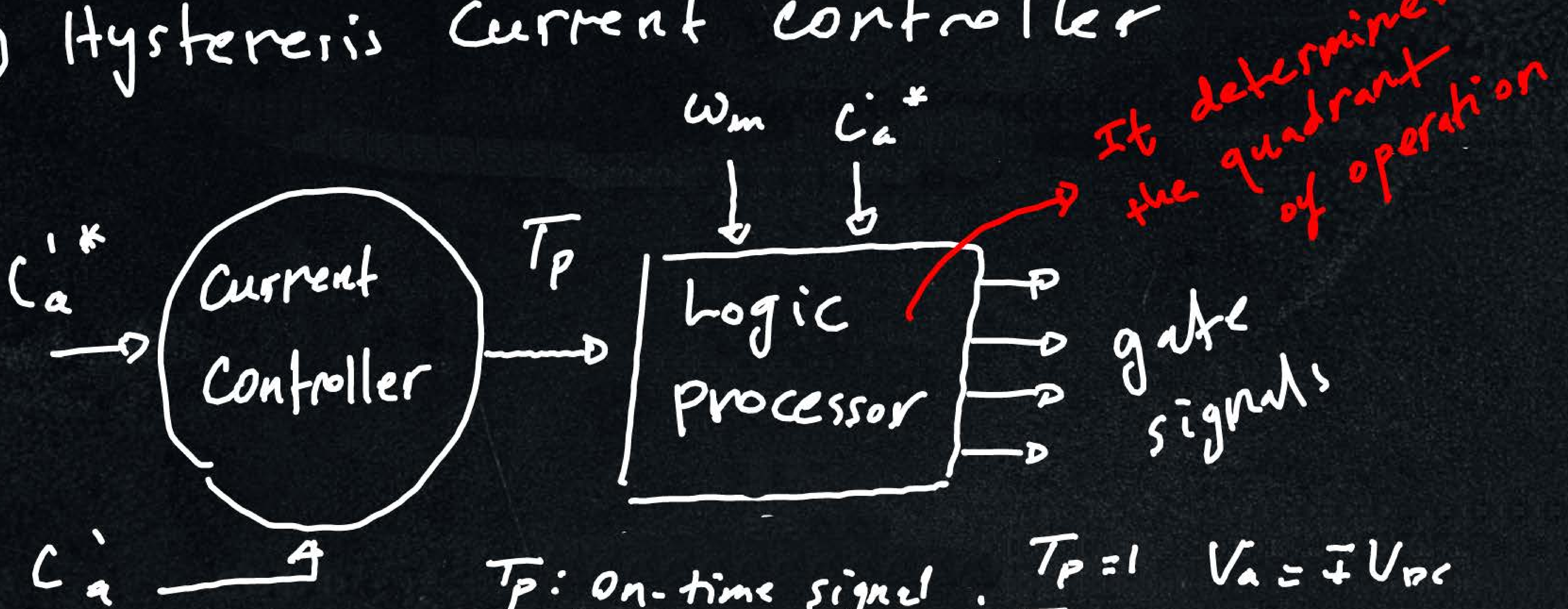
$$\boxed{\omega_{m,ss} = A \longrightarrow \text{zero ss error}}$$



Current Controller & logic processor

① PWM current controller

② Hysteresis current controller



T_p: On-time signal. $T_p = 1$ $V_a = \mp V_{DC}$

$T_p = 0$ $V_a = 0$

Table of logic processor

Q	T_p	T_1	T_2	T_3	T_4
I	1	1	1	0	0
I	0	1	0	0	0
II	1	0	0	0	0
II	0	1	0	0	0
III	1	0	0	1	1
III	0	0	0	1	0
IV	1	0	0	0	0
IV	0	0	0	1	0

1 : ON

0 : OFF

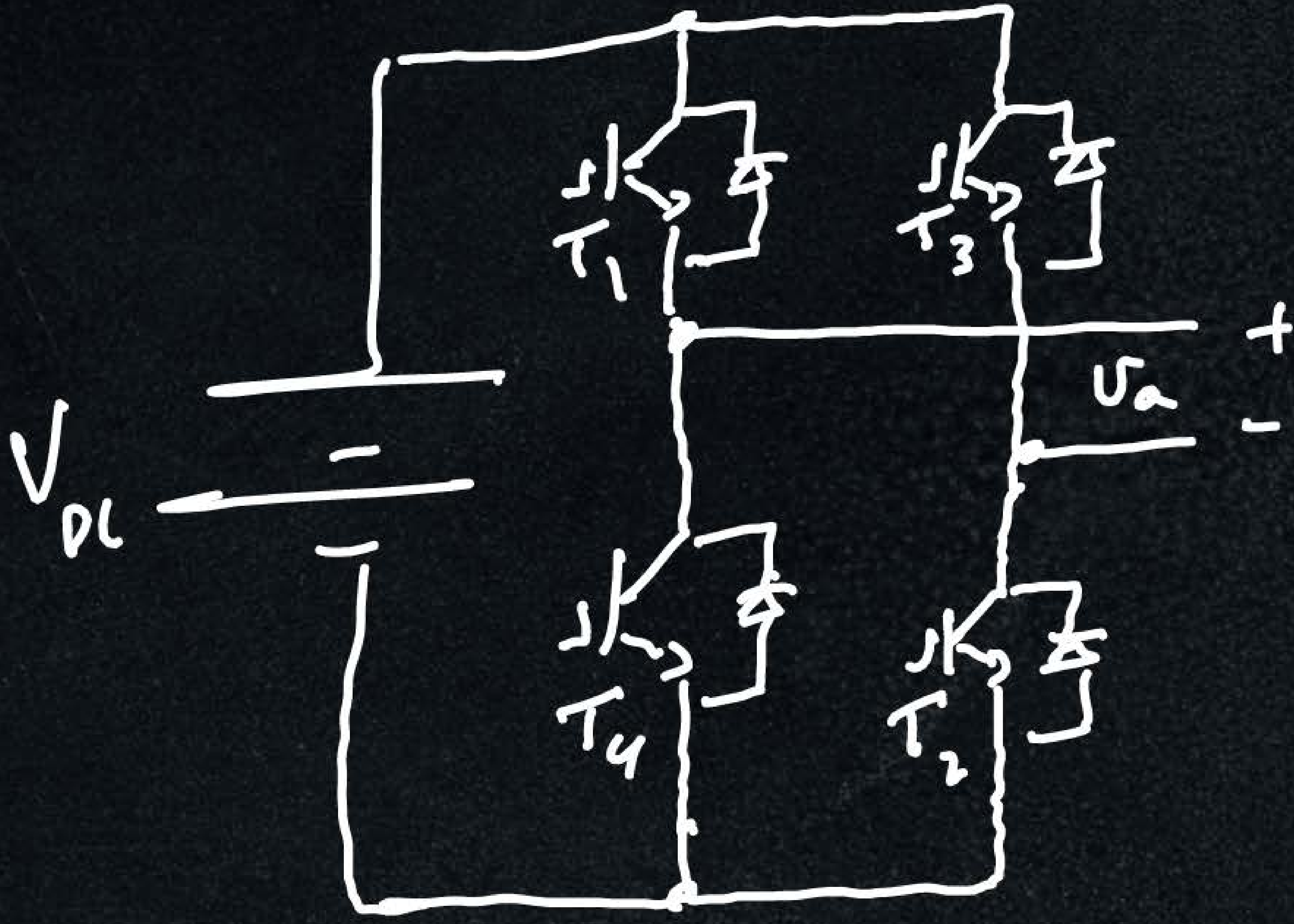


Table of logic processor

Two-quadrant chopper

Q	Tp	T_1 T_2
I	1	1 0
I	0	0 0
IV	1	0 0
IV	0	0 1

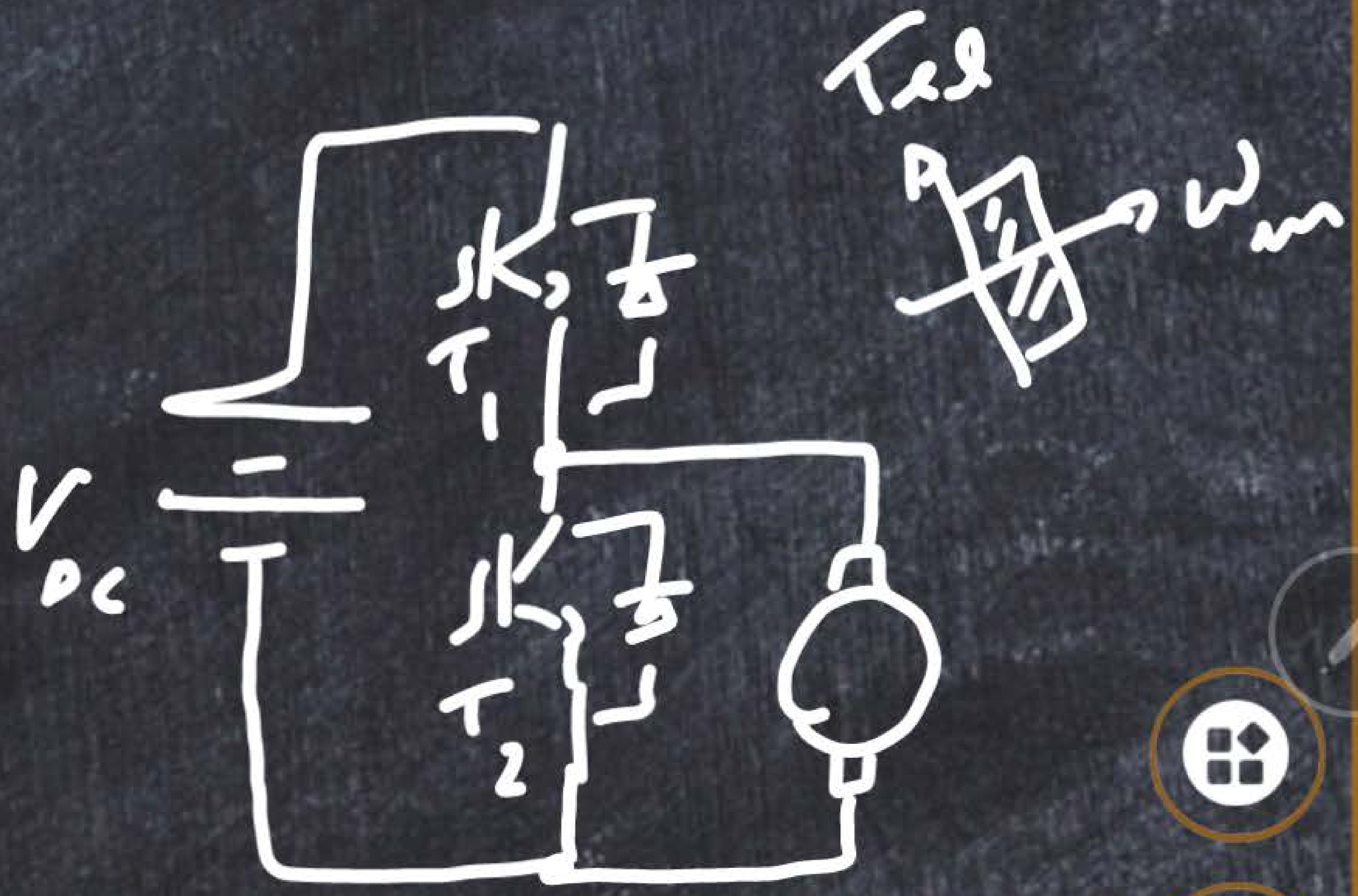
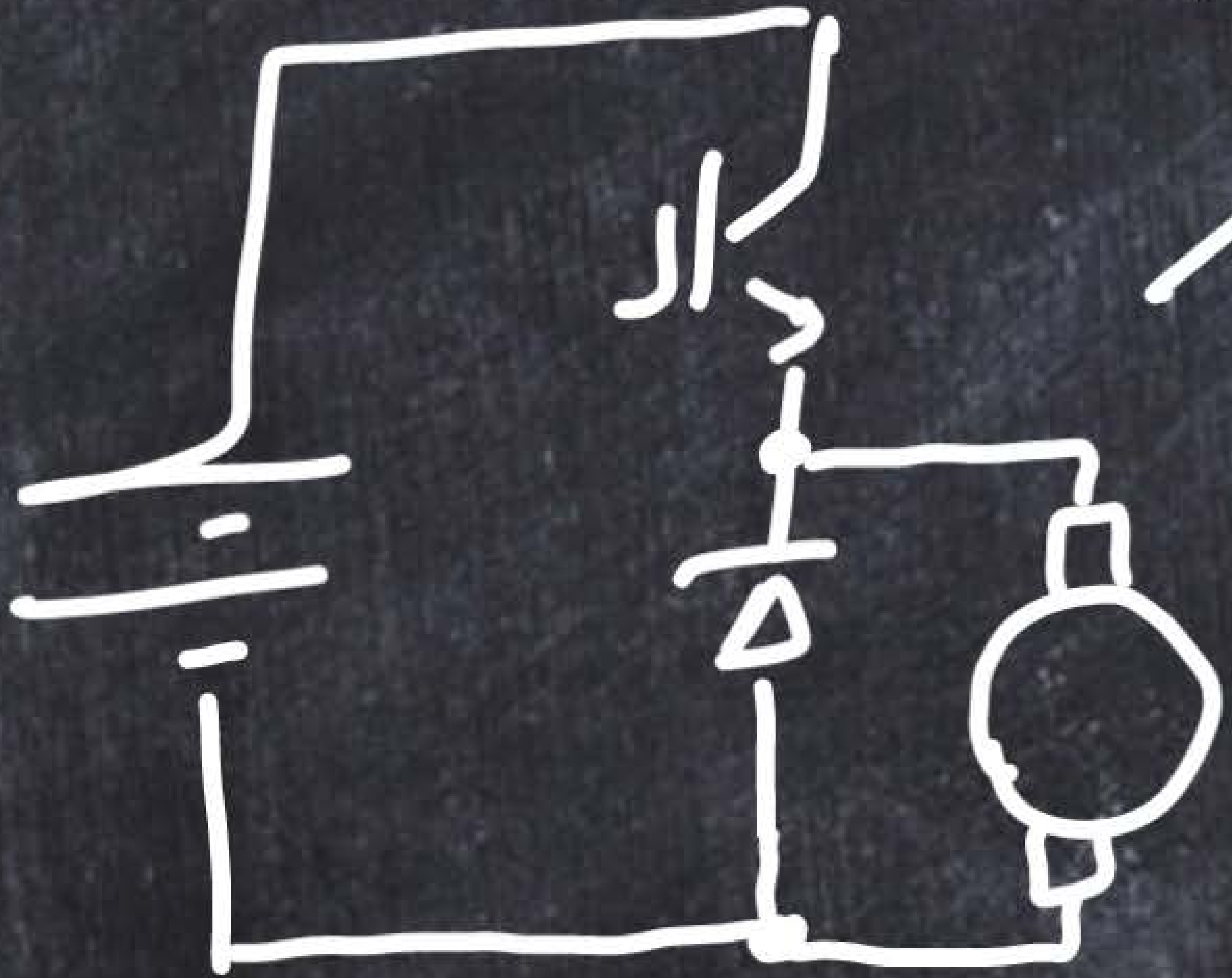
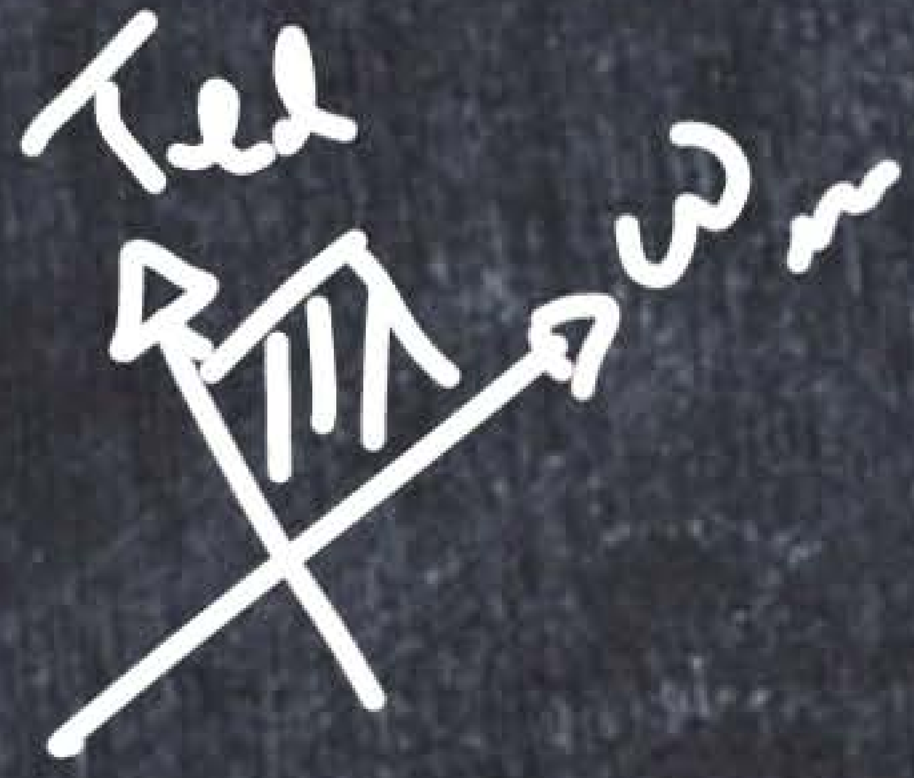


Table of logic processor

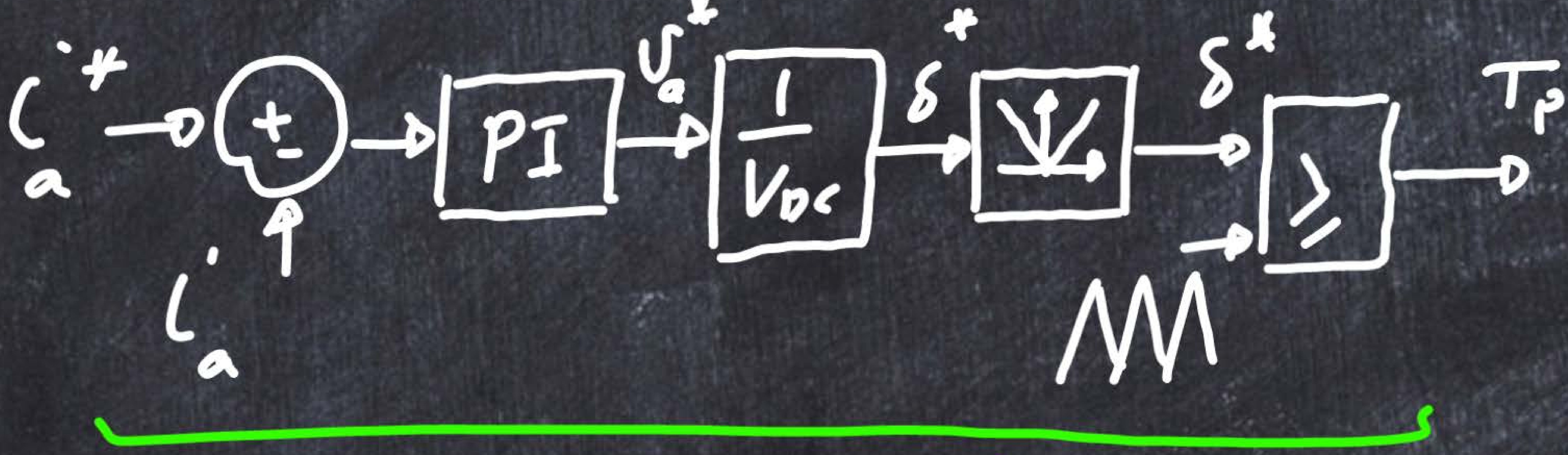
One-quadrant chopper

Q	T_p	T_1
I	1	1
I	0	0





① PWM current controller



Impelementation of PWM CC

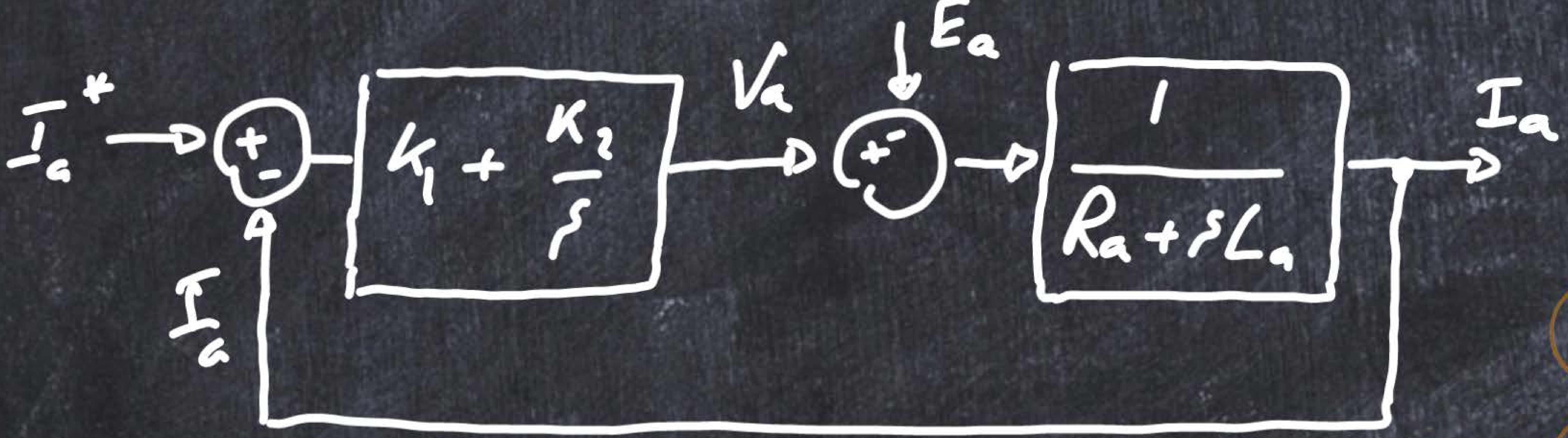
Design of PWM CC

It is designed by using the KVL equation in the armature circuit:

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a$$

$$\xrightarrow{\mathcal{L}} V_a = (R_a + sL_a) I_a + E_a$$

Block diagram in s-domain



$$I_a = T_1 I_a^* + T_2 E_a$$

Transfer function

$$I_a = T_1 I_a^* + T_2 E_a$$

$$T_1 = \left.\frac{I_a}{I_a^*}\right|_{E_a = 0} = \frac{G}{1+G} \; ; \; G = \frac{K_1 s + K_2}{s[R_a + sL_a]}$$

$$T_1 = \frac{(K_1 s + K_2)/(s(R_a + sL_a))}{1 + (K_1 s + K_2)/(s(R_a + sL_a))}$$

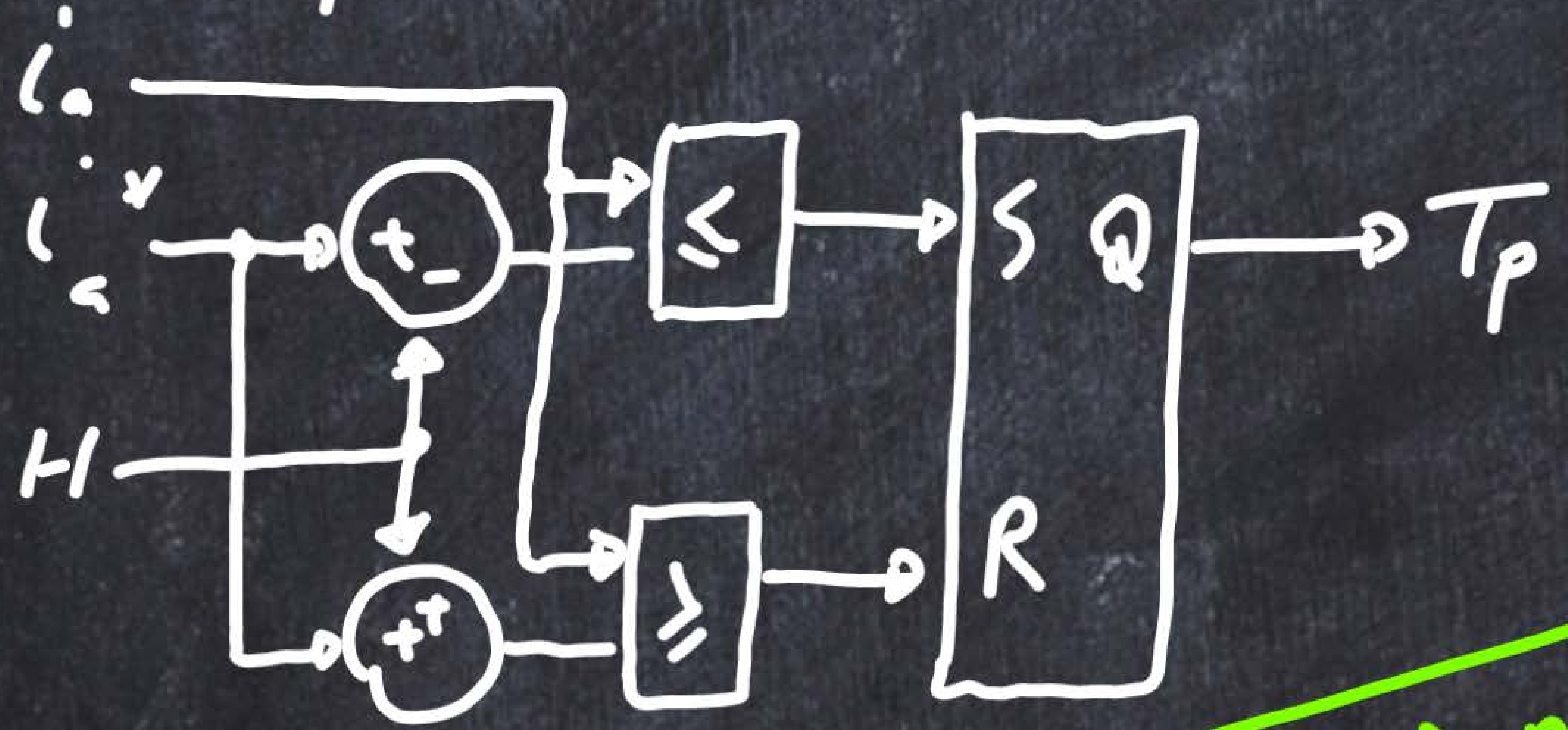
$$T_1 = \frac{K_1 s + K_2}{K_1 s + K_2 + s^2 L_a + R_a s} = \frac{K_1 s + K_2}{s^2 L_a + (K_1 + R_a) s + K_2}$$

$$T_1 = \frac{K_1}{L_a}\left(\frac{s + K_2/K_1}{\underbrace{s^2 + \left[(K_1 + R_a)/L_a\right] s + K_2/L_a}_{s^2 + 2\zeta\omega_n s + \omega_n^2}}\right)$$

$$K_1 = 2\zeta\omega_n L_a - R_a$$

$$K_2 = \omega_n^2 L_a$$

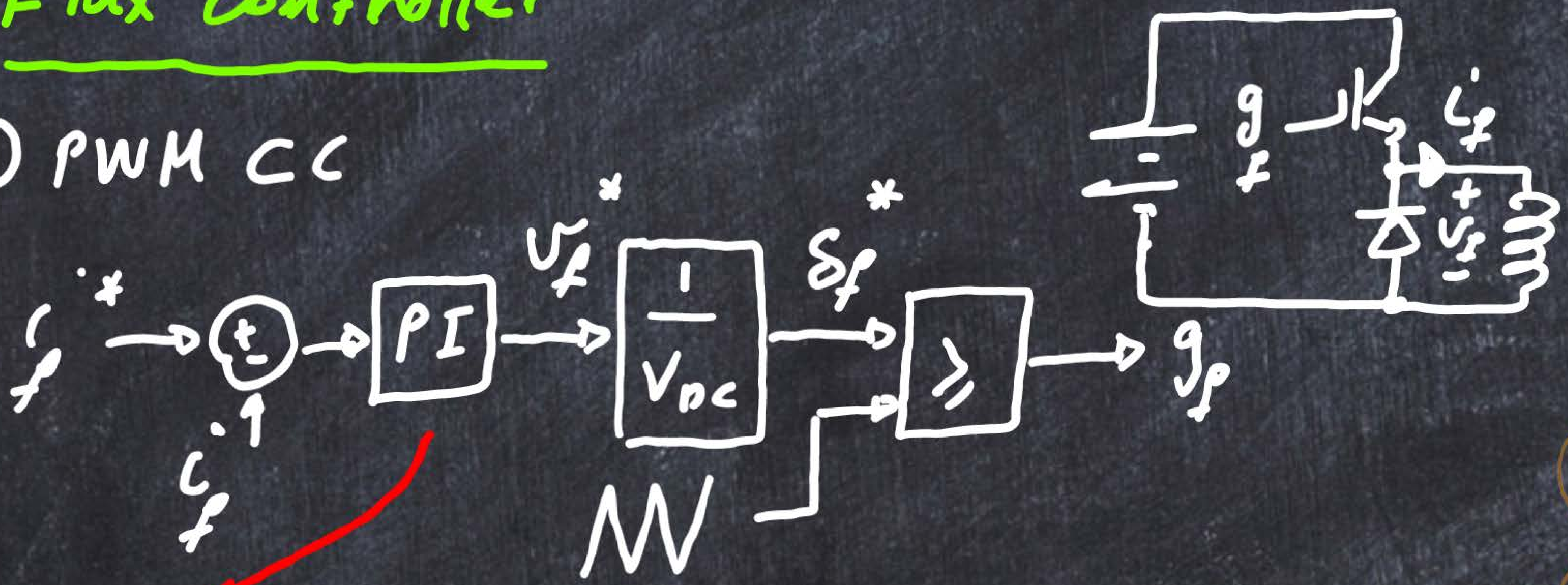
② Hysteresis Current Controller



Implementation of HCC

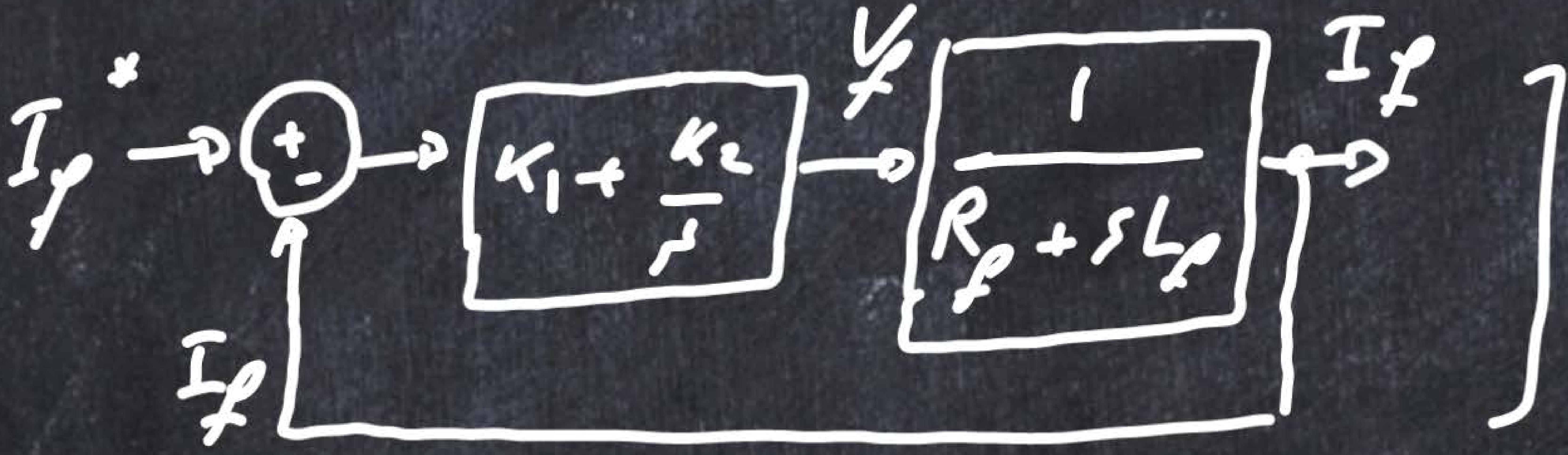
Flux controller

① PWM CC



It is designed using the KVL equation in the field circuit

$$v_f = R_f + L\frac{di_f}{dt} \xrightarrow{\mathcal{L}} V_f = (R_f + sL_f)\, I_f$$



Block diagram

$$I_f = T\, I_f^* \;;\; T = \frac{G}{1+G} \;;\; G = \frac{K_1 s + K_2}{s[R_f + sL_f]}$$

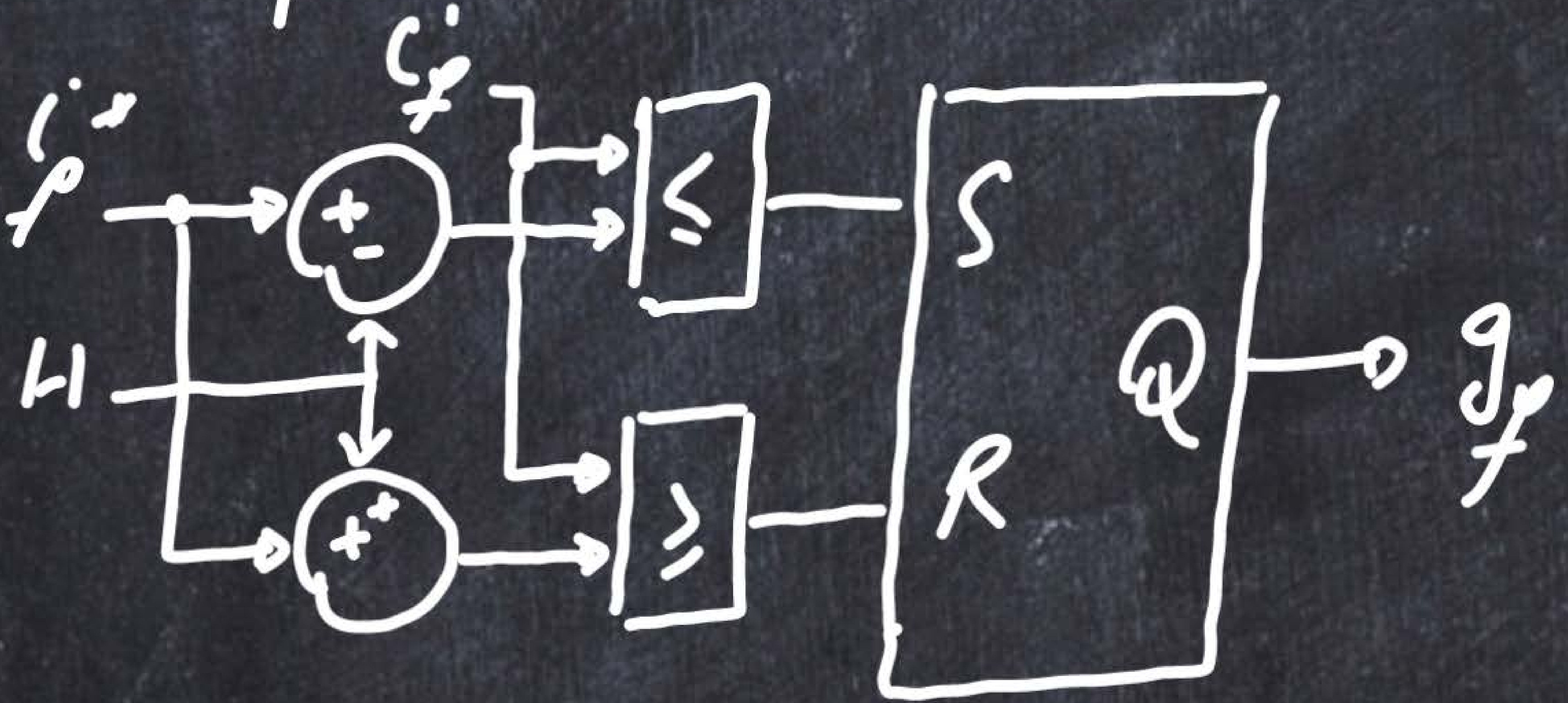
$$T = \frac{K_1 s + K_2}{K_1 s + K_2 + R_f s + s^2 L_f}$$

$$T = \frac{K_1}{L_f}\left[\frac{s + \frac{K_2}{K_1}}{\underbrace{s^2 + \left(\frac{K_1 + R_f}{L_f}\right)s + \frac{K_2}{L_f}}_{s^2 + 2\zeta\omega_n s + \omega_n^2}}\right]$$

$$K_1 = 2\zeta\omega_n L_f - R_f$$

$$K_2 = \omega_n^2 L_f$$

② Hysteresis CC



Implementation

DC Motor Drive with Controlled Rectifiers

① Single-phase Controlled Rectifiers

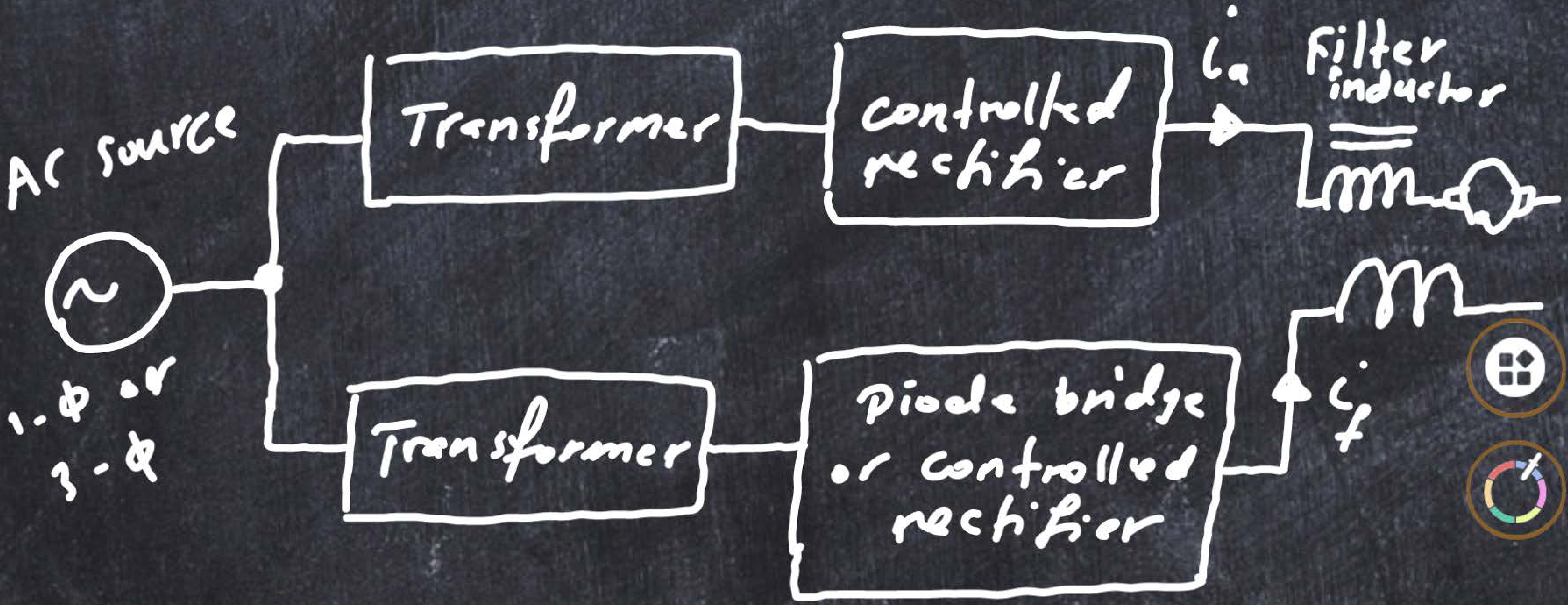
1.1) Half-wave

1.2) Semiconverter

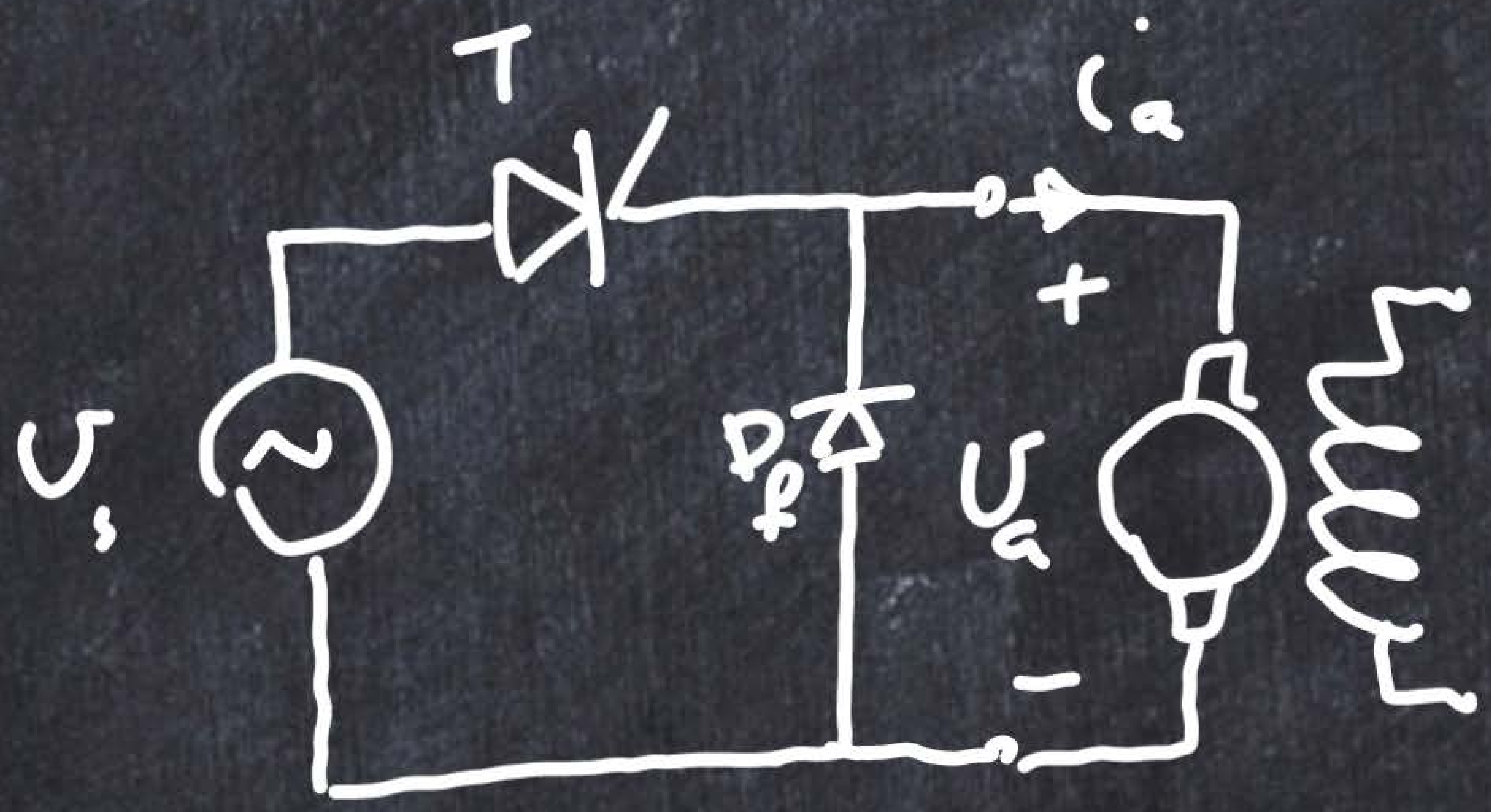
1.3) Full-wave

1.4) Dual converter

Line Diagram



1.1) Half wave



$$V_s = V_m \sin(\omega t)$$

1.1) Half_wave

$$V_a = \frac{1}{2\pi} \int_0^{2\pi} v_a \, d(\omega t)$$

$$V_a = \frac{1}{2\pi} \int_\alpha^{\pi} V_m \sin(\omega t) \, d\omega t = \frac{V_m}{2\pi} \left[\cos \omega t \Big|_\pi^\alpha \right]$$

$$\boxed{V_a = \frac{V_m}{2\pi} (1 + \cos\alpha)}$$

1.2) Semiconverter

$$V_a = \frac{1}{2\pi}\int_0^{2\pi} v_a \, d(\omega t) = \frac{2}{2\pi}\int_\alpha^{\pi} V_m \sin(\omega t)\, d\omega t$$

$$V_a = \frac{V_m}{\pi}\cos(\omega t)\Big|_{\pi}^{\alpha}$$

$$\boxed{V_a = \frac{V_m}{\pi}(1+\cos\alpha)}$$

1.3) Full-wave

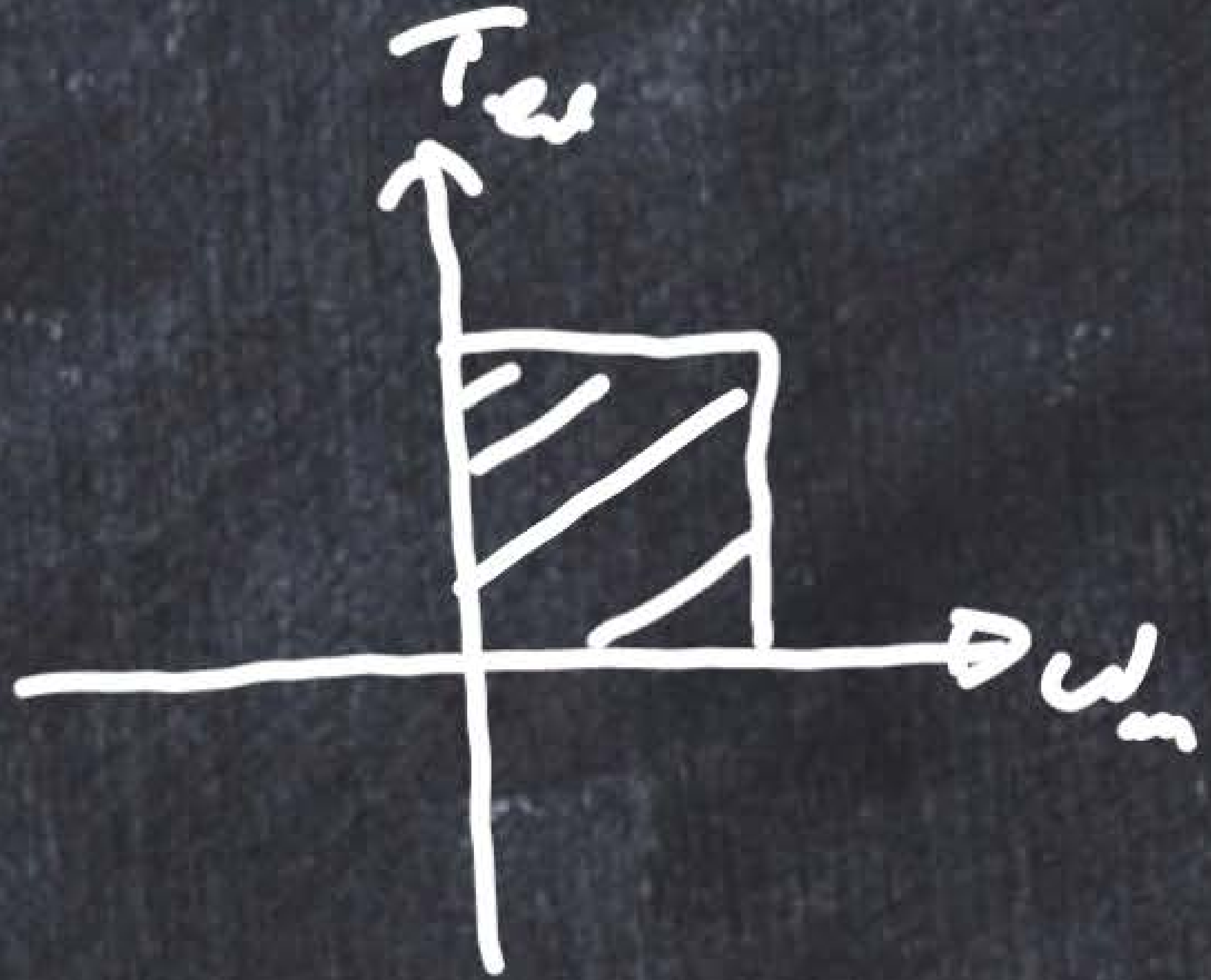
$$V_a = \frac{1}{2\pi}\int_0^{2\pi} V_a \, d(\omega t) = \frac{2}{2\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t)\, d\omega t$$

$$V_a = \frac{V_m}{\pi} \cos\omega t \Big|_{\pi+\alpha}^{\alpha}$$

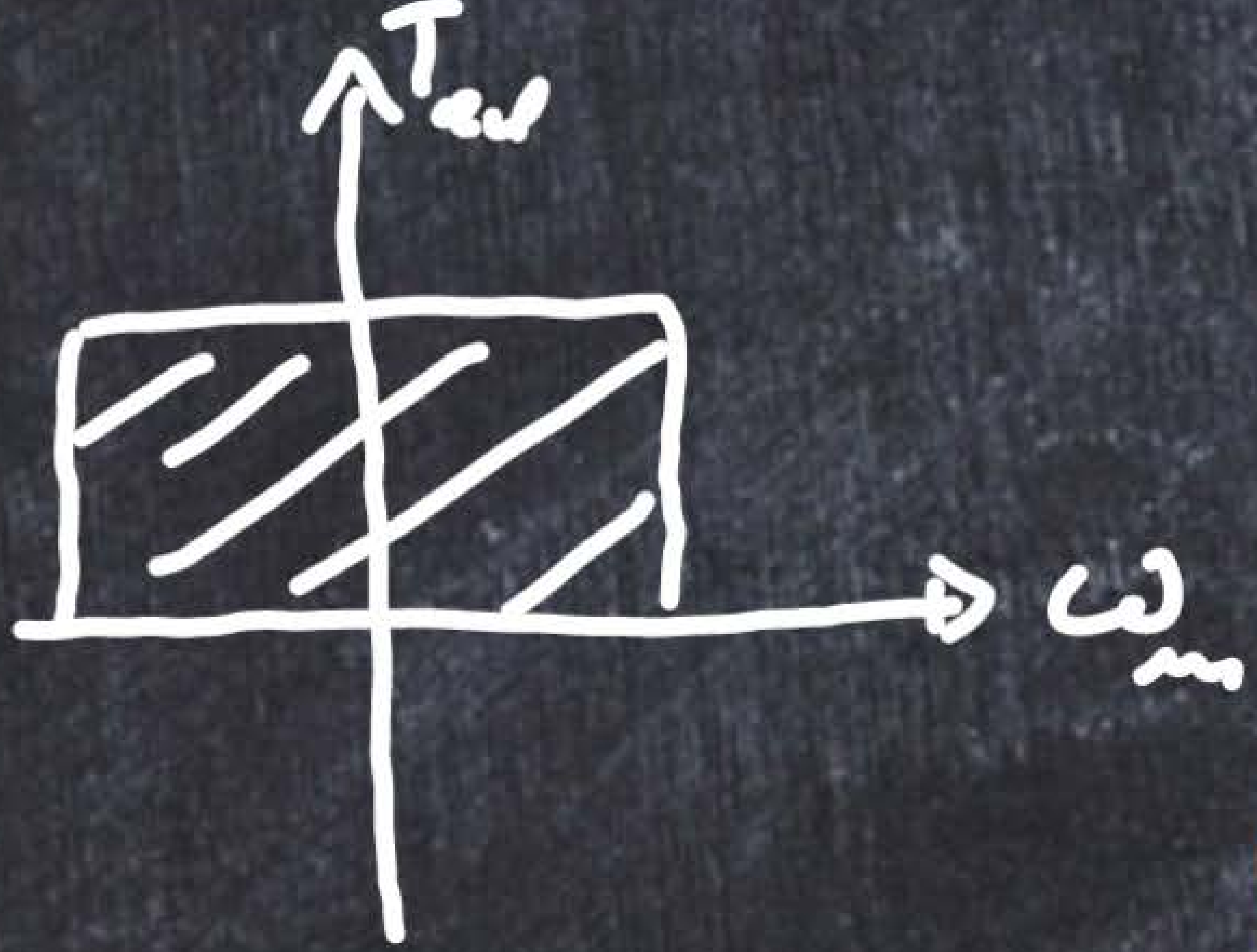
$$\boxed{V_a = \frac{2V_m}{\pi}\cos\alpha}$$

Half

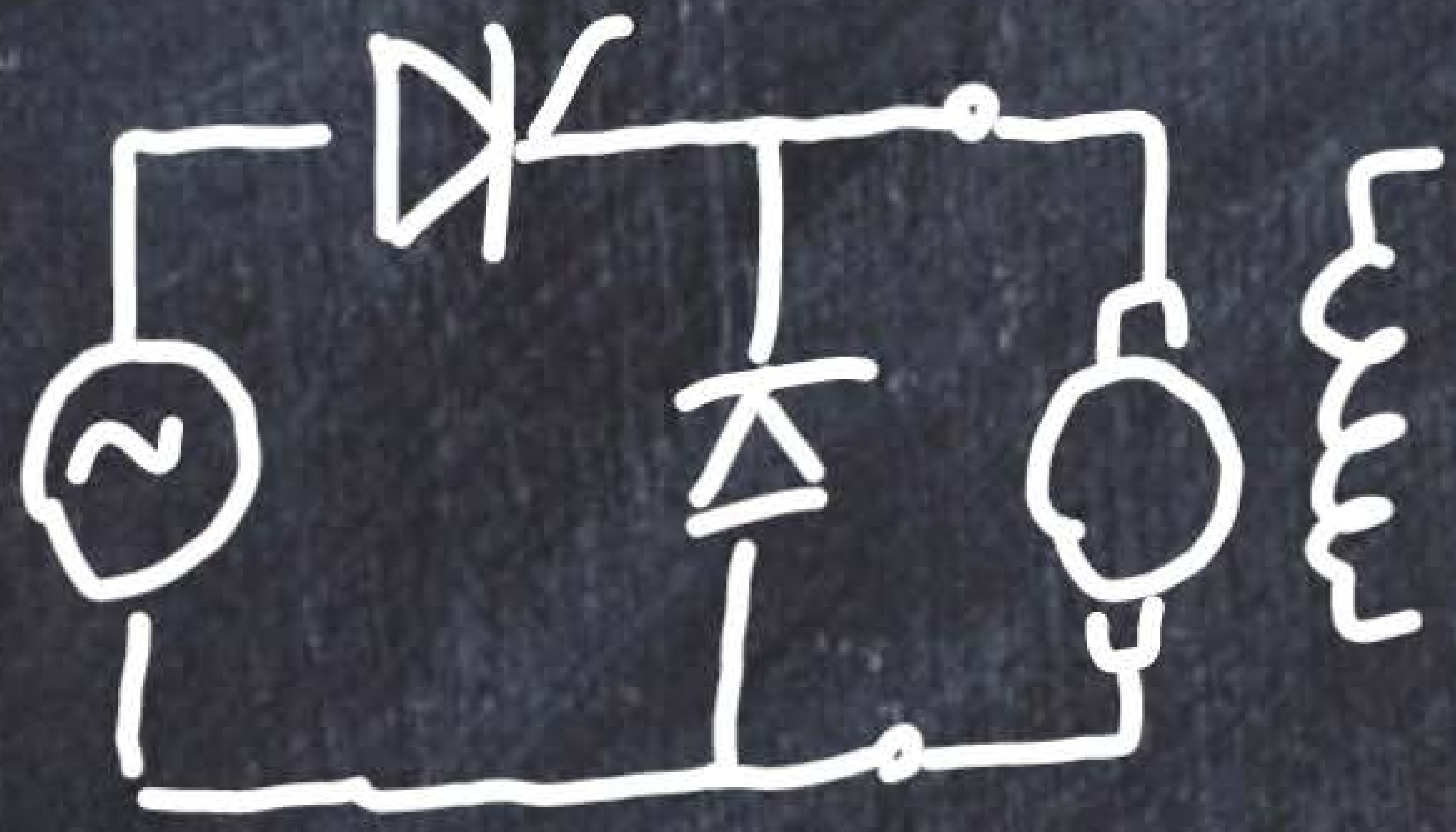
Semi



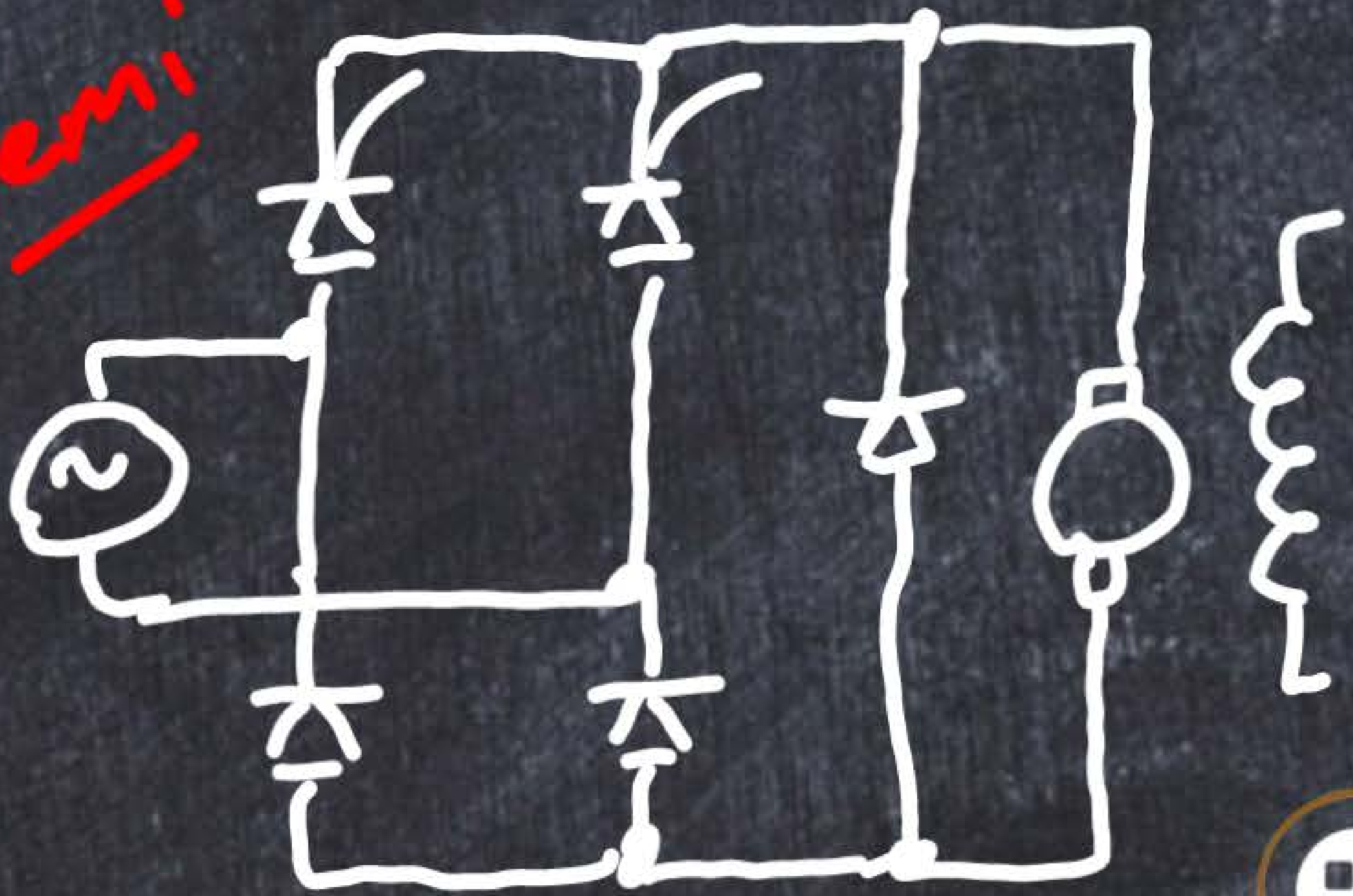
Full



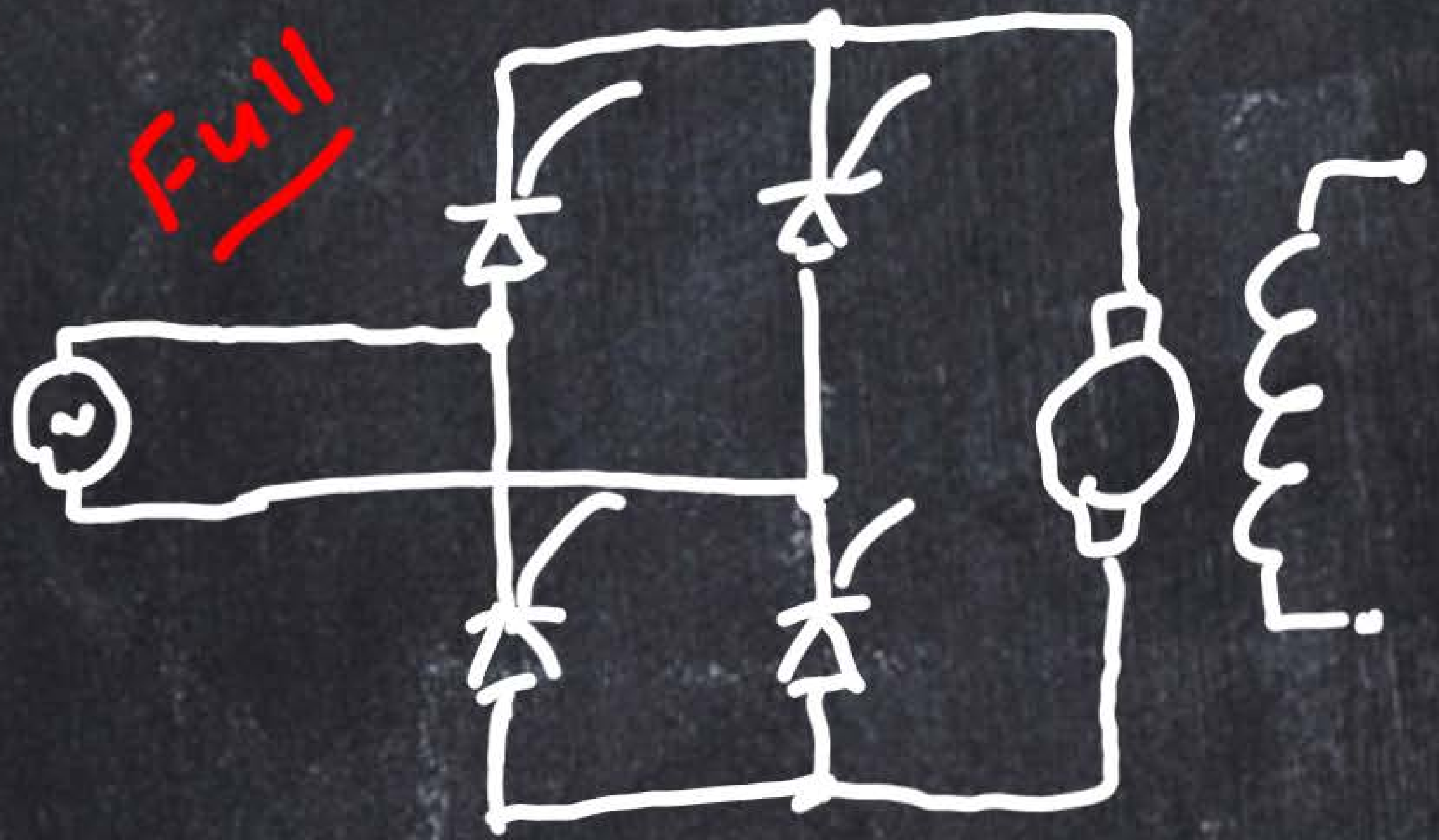
Half



Semi



Full



② Three-Phase controlled rectifiers

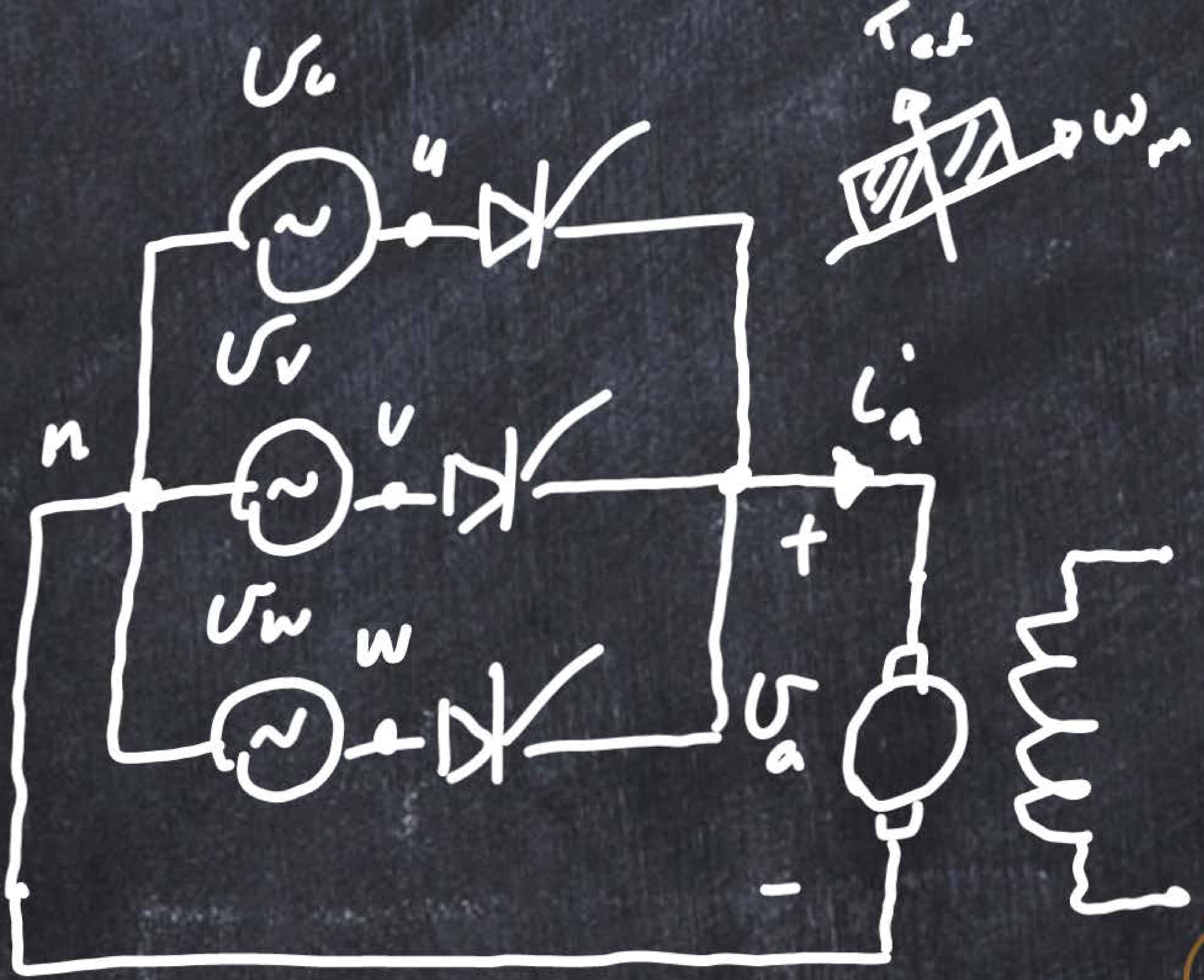
2.1) Half wave

2.2) semiconverter

2.3) Full wave

2.4) Dual converter

2.1) Half wave



$$V_u = V_m \sin(\omega t)$$
$$V_v = V_m \sin(\omega t - \tfrac{2\pi}{3})$$
$$V_w = V_m \sin(\omega t + \tfrac{2\pi}{3})$$

phase voltages

$$V_a = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin(\omega t)\, d(\omega t)$$

$$= \frac{-3V_m}{2\pi} \cos(\omega t) \Big|_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}$$

$$= \frac{3V_m}{2\pi}\left[\cos\left(\pi/6+\alpha\right) - \cos\left(\frac{5\pi}{6}+\alpha\right)\right]$$

$$V_a = \frac{3V_m}{2\pi}\left[\left(\cos\frac{\pi}{6}\cos\alpha - \sin\frac{\pi}{6}\sin\alpha\right) - \left(\cos\frac{5\pi}{6}\cos\alpha - \sin\frac{5\pi}{6}\sin\alpha\right)\right]$$

$$V_a = \frac{3V_m}{2\pi}\left[\left(\frac{\sqrt{3}}{2}\cos\alpha - \cancel{\frac{1}{2}\sin\alpha}\right) - \left(-\frac{\sqrt{3}}{2}\cos\alpha - \cancel{\frac{1}{2}\sin\alpha}\right)\right]$$

$$\boxed{V_a = \frac{3\sqrt{3}\,V_m}{2\pi}\cos\alpha}$$

$$-\frac{3\sqrt{3}\,V_m}{2\pi} \leq V_a \leq \frac{3\sqrt{3}\,V_m}{2\pi}$$

$$V_a = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\pi/2} v_{uv}\, d\omega t + \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} v_{uw}\, d\omega t\right]$$

v_{uv}

v_{vw}

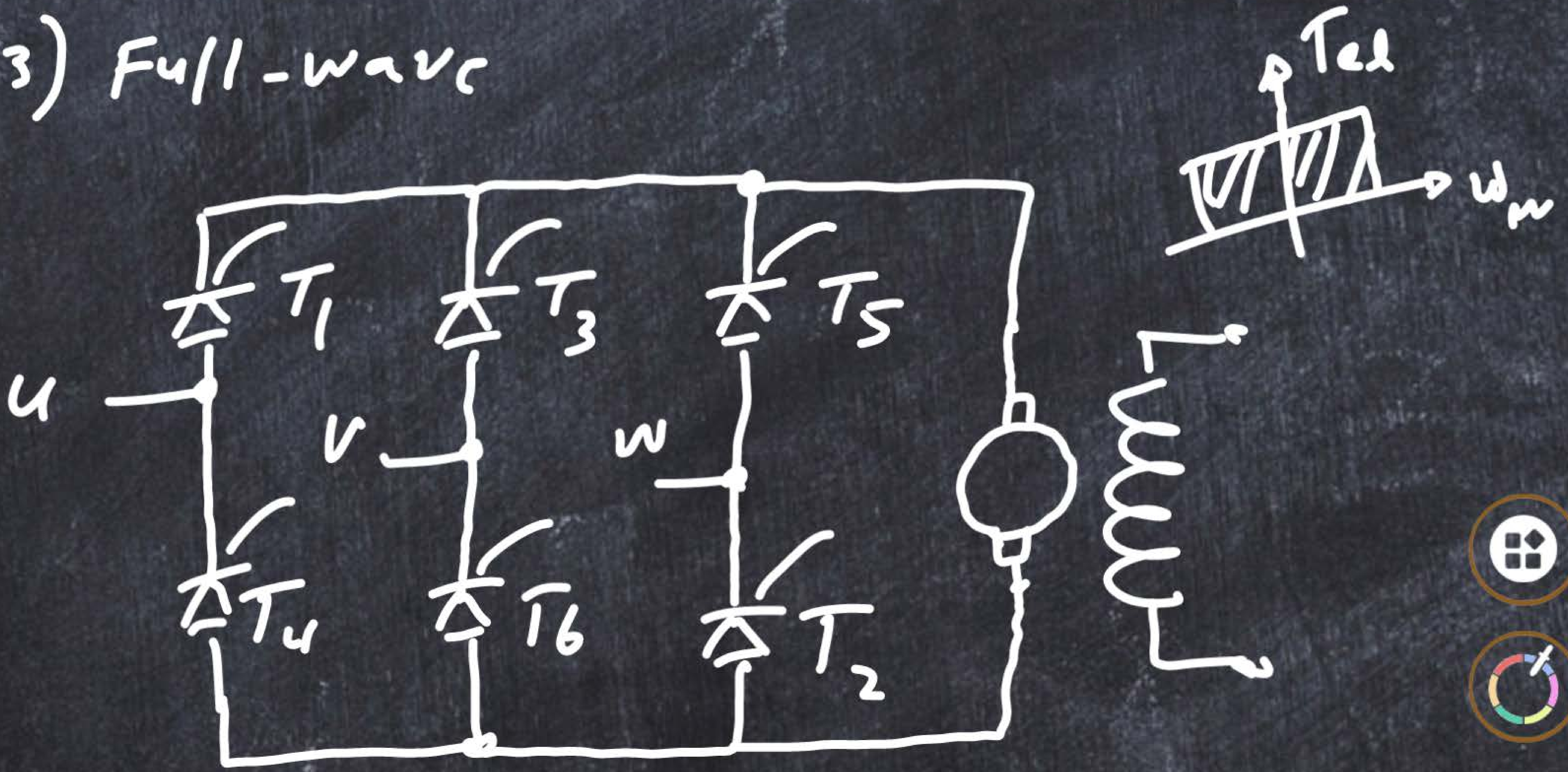
v_{wu}

$$v_{uv} = \sqrt{3}\, V_m \sin(\omega t + 30^\circ)$$

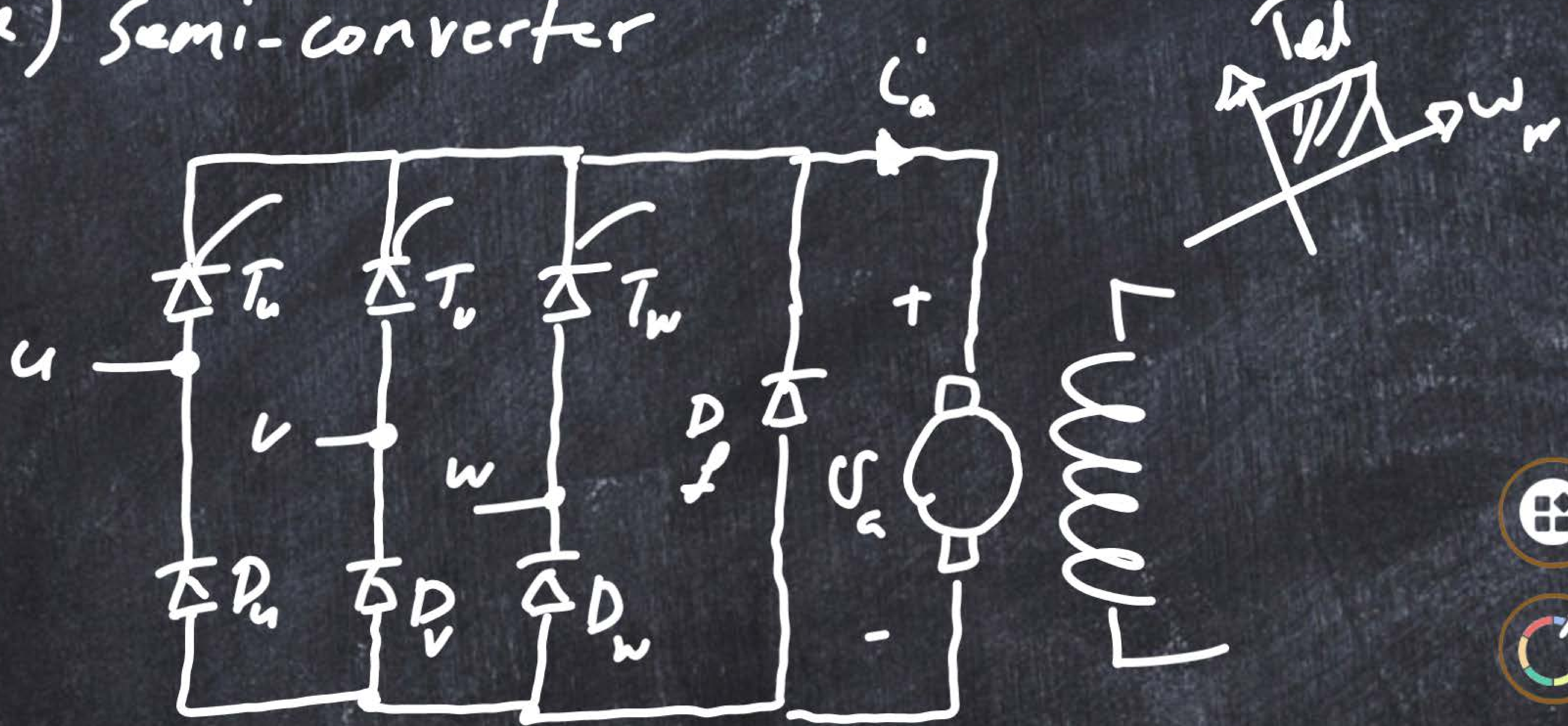
$$v_{uw} = -\sqrt{3}\, V_m \sin(\omega t + 30 + 120^\circ)$$

$$\boxed{V_a = \frac{3\sqrt{3}\, V_m}{2\pi}(1 + \cos\alpha)}$$

2.3) Full-wave



2.2) Semi-converter



$$V_a = \frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi/2+\alpha} v_{uv}\, d\omega t \quad ; \quad v_{uv} = \sqrt{3}\, V_m \sin(\omega t + 30^\circ)$$

$$\boxed{V_a = \frac{3\sqrt{3}\, V_m}{\pi}\cos\alpha}$$

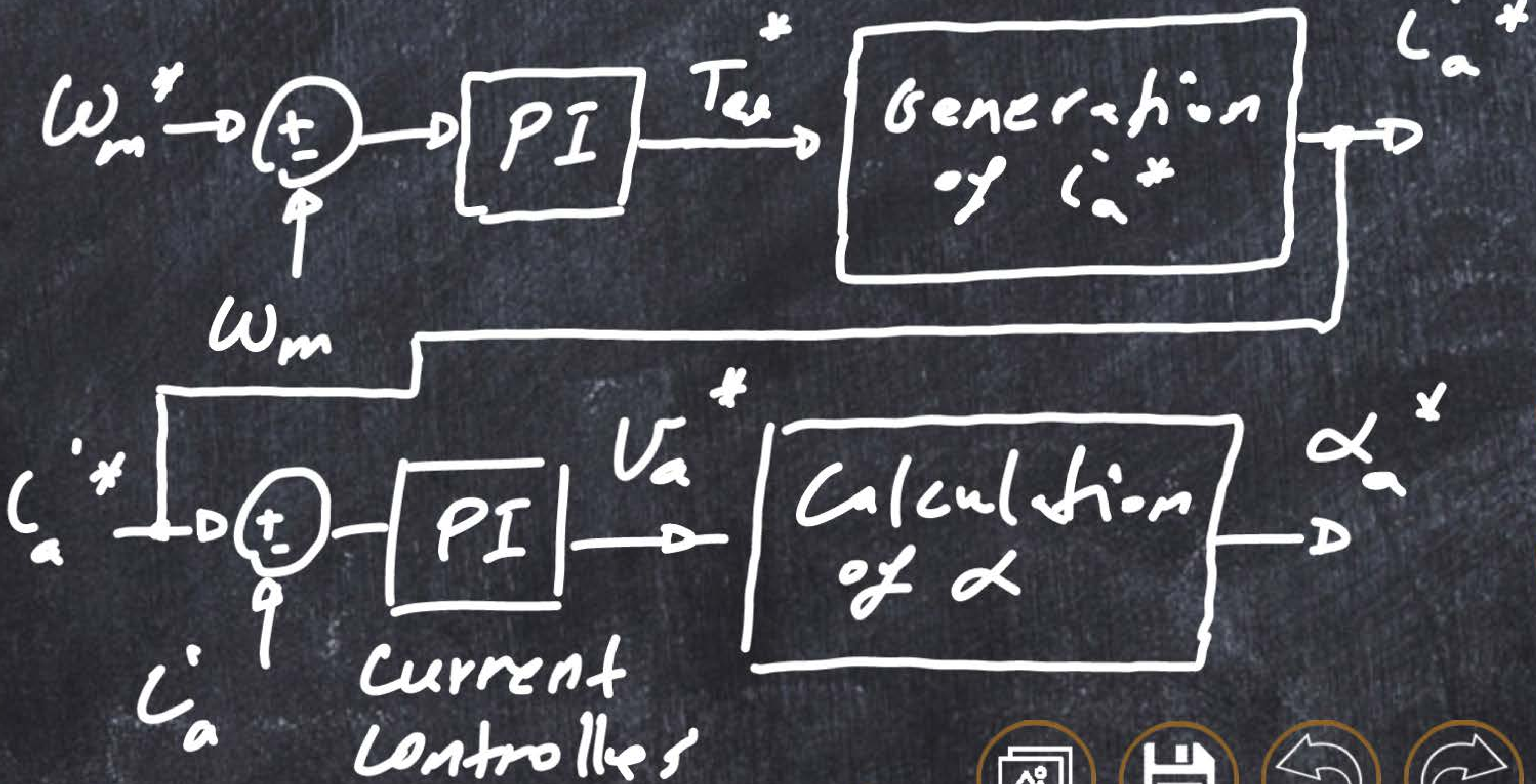
$1-\phi$

Circuit	V_a
Half	$\frac{V_m}{2\pi}(1+\cos\alpha)$
Semi	$\frac{V_m}{\pi}(1+\cos\alpha)$
Full	$\frac{2V_m}{\pi}\cos\alpha$

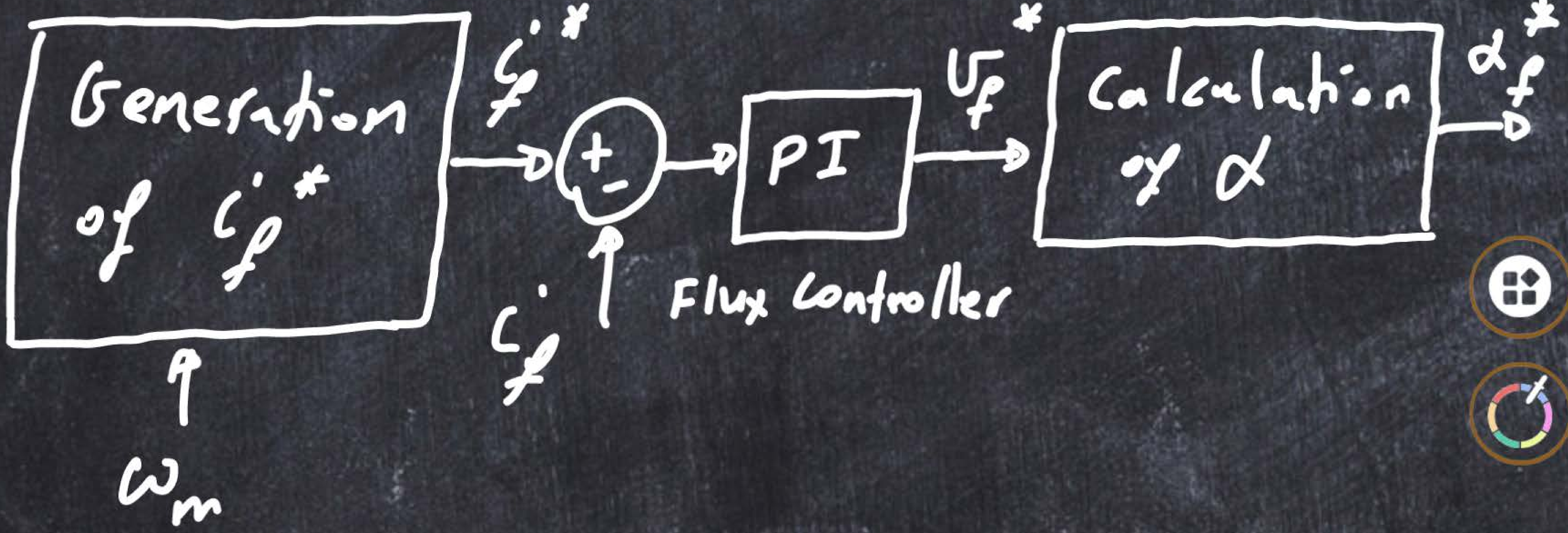
3-Φ

Circuit	V_a
Half	$\dfrac{3\sqrt{3}\,V_m}{2\pi}\cos\alpha$
Semi	$\dfrac{3\sqrt{3}\,V_m}{2\pi}(1+\cos\alpha)$
Full	$\dfrac{3\sqrt{3}\,V_m}{\pi}\cos\alpha$

Control system "Armature circuit"



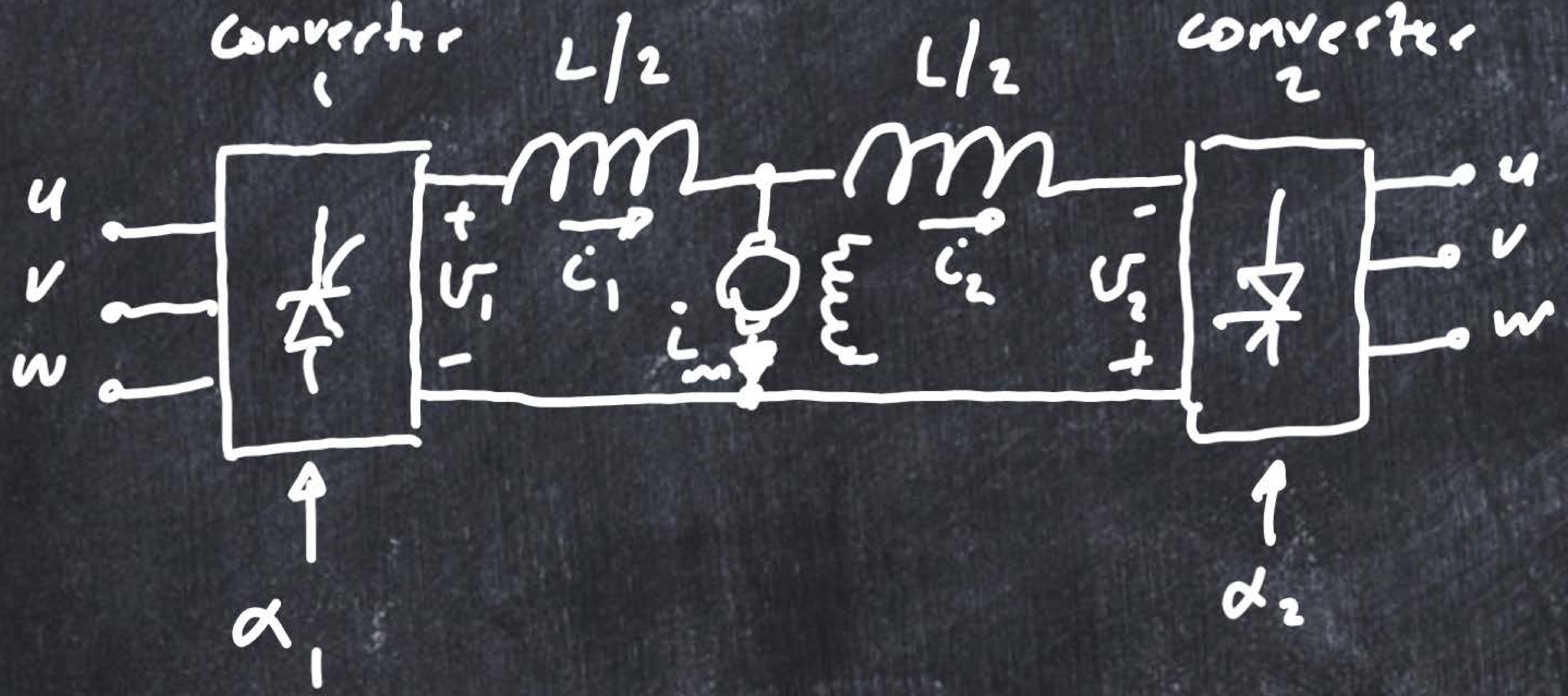
Control System "Field circuit"



Dual Converter

- The Controlled rectifier is voltage reversible, but not current reversible.
- To obtain 4-quadrant operation, it is necessary to have two converters operating back-to-back or in anti-parallel
- The converter can be controlled in 2 ways:
  - circulating current method
  - Non-circulating current method

Circulating current scheme



- The converters are controlled to produce the same DC voltage across the motor terminals:

$$-v_1 + \frac{L}{2}\frac{di_1}{dt} + \frac{L}{2}\frac{di_2}{dt} - v_2 = 0$$

$$-V_1 - V_2 = 0 \qquad V_1 = \frac{1}{T}\int_0^T v_1\,dt \,, \; V_2 = \frac{1}{T}\int_0^T v_2\,dt$$

$$V_1 + V_2 = 0$$

$$\cos\alpha_1 + \cos\alpha_2 = 0$$

$$\cos\alpha_1 = -\cos\alpha_2 = \cos(\pi - \alpha_2)$$

$$\Rightarrow \alpha_1 = \pi - \alpha_2$$

or $$\boxed{\alpha_1 + \alpha_2 = \pi}$$

$$i_{cir} = i_{cir,DC} + i_{cir,AC}$$

$$i_{cir,DC} = 0 \quad \text{since} \quad V_1 = V_2$$

$$i_{cir,AC} \neq 0 \quad \text{since} \quad v_1 \neq v_2$$

Note: i_{cir} flows only in one direction.

- $i'_{cir,AC}$ is limited by using buffer inductor, L.

- KVL in the loop: $v_1 + v_2 = \frac{L}{2}\frac{di_1}{dt} + \frac{L}{2}\frac{di_2}{dt}$

$i_1 = i'_m + i'_{cir,AC}$

$i_2 = i'_{cir,AC}$

$i'_1 = i'_{cir,AC}$

$i'_2 = i'_m + i'_{cir,AC}$

$$v_1 + v_2 = L \frac{di_{cir,AC}}{dt} \Rightarrow i_{cir,AC} = \frac{1}{L}\int_0^t (v_1 + v_2)\,dt$$

$i_{cir,AC}$

$$0 \le \alpha_1 \le \frac{\pi}{3}$$

$i_{cir,AC}$

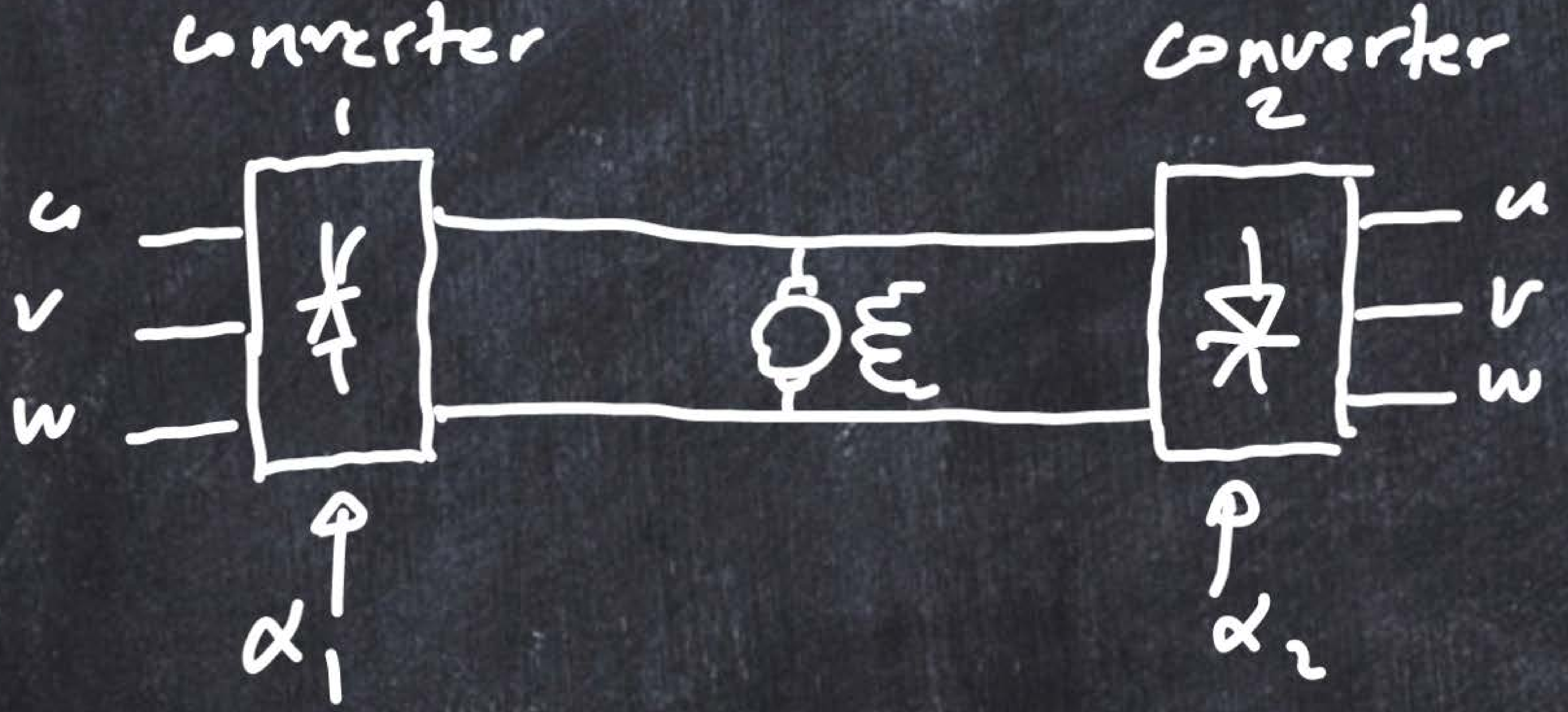
$$\frac{\pi}{3} \le \alpha_1 \le \frac{2\pi}{3}$$

3-ϕ half-Wave

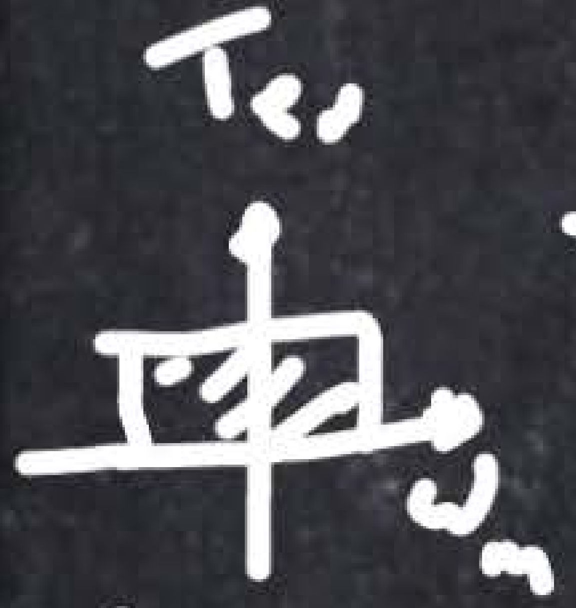
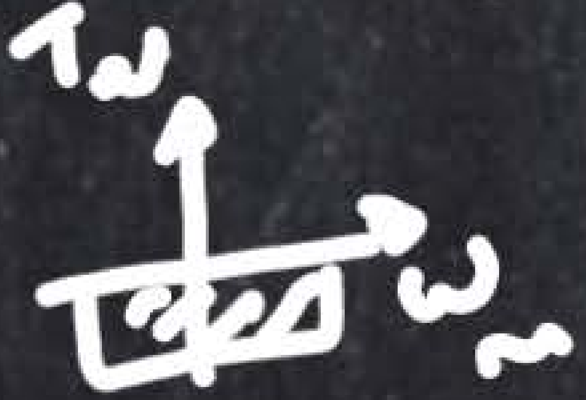
→ In order for the combination to respond rapidly to control signals, it is necessary for SCRs to be in contineous conduction. This can be achieved by making $\alpha_1 + \alpha_2$ is very slightly less than π, which effectively creats $i'_{cir,DC}$ component, causing i'_{cir} to be all the time contineous.

The main disadvantage of this scheme is the use of inductors, which are larger and more expensive.

Non-circulating current scheme



→ In this scheme, only one converter recieves firing pulses at a time.

	i_m^*	$i_1^* =$	$i_2^* =$	converter 1	converter 2
	+	i_m^*	0	ON	OFF
	−	0	i_m^*	OFF	ON